

# Department of Economics

## Testing the Martingale Difference Hypothesis Using Neural Network Approximations

George Kapetanios and Andrew P. Blake

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# Testing the martingale difference hypothesis using neural network approximations

George Kapetanios\*  
Queen Mary, University of London

Andrew P. Blake†  
Bank of England

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## Abstract

The martingale difference restriction is an outcome of many theoretical analyses in economics and finance. A large body of econometric literature deals with tests of that restriction. We provide new tests based on radial basis function neural networks. Our work is based on the test design of Blake and Kapetanios (2000, 2003a,b). However, unlike that work we can provide a formal theoretical justification for the validity of these tests using approximation results from Kapetanios and Blake (2007). These results take advantage of the link between the algorithms of Blake and Kapetanios (2000, 2003a,b) and boosting. We carry out a Monte Carlo study of the properties of the new tests and find that they have superior power performance to all existing tests of the martingale difference hypothesis we consider. An empirical application to the S&P500 constituents illustrates the usefulness of our new test.

*Keywords:* Martingale Difference Hypothesis, Neural Networks, Boosting.

*JEL Classification:* C14.

## 1 Introduction

The martingale or martingale difference restriction arises repeatedly in finance and economics. Rational expectations, market efficiency and similar theoretical frameworks impose this restriction on economic variables such as consumption and stock returns. From an econometric point of view, the martingale difference hypothesis (MDH) amounts to the statement that the best linear predictor of a covariance stationary stochastic process, at any point in time, conditional on the currently available information set, is equal to the unconditional expectation. It is useful to have tests for this restriction as tools for falsifying economic theories.

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\*Department of Economics, Queen Mary, University of London, Mile End Road, London E1 4NS, UK. Email: G.Kapetanios@qmul.ac.uk.

†Monetary Analysis, Bank of England, Threadneedle Street, London EC2R 8AH, UK. Email: Andrew.Blake@bankofengland.co.uk.

A number of such tests have been proposed in the literature. Bierens and Ploberger (1991) provided a test based on the fact that under the MDH the spectral distribution function is a straight line. Deo (2000) has provided extensions of this test to conditional heteroscedasticity. As noted by Escanciano and Velasco (2007a), the test based on the spectral distribution function is not consistent against all deviations from MDH and, in particular, it cannot detect deviations that imply lack of autocorrelation. Escanciano and Velasco (2007a) proposes a new test based on the fact that, for a process  $y_t$  that satisfies the MDH,

$$E(y_t|I_{t-1}) = \delta \quad a.s. \Leftrightarrow E((y_t - \delta)\mu(I_{t-1})) = 0 \quad (1)$$

for some constant  $\delta$  and any  $\mathcal{F}_{t-1}$  measurable function  $\mu(\cdot)$ , where  $I_{t-1} = (y_{t-1}, y_{t-2}, \dots)'$ ,  $\mathcal{F}_t$  is the  $\sigma$ -field generated by  $I_{t-1}$ . Noting the equivalence in (1), links the MDH testing problem to a large specification testing literature that aims to capture deviations from some parametric null hypothesis and uses tests based on particular forms for  $\mu(\cdot)$  to do so. The most popular forms for  $\mu(\cdot)$  are the exponential function used by Bierens (1984, 1990); Bierens and Ploberger (1997); Hong (1999a,b); de Jong (1996) and the indicator function used by Stute (1997); Koul and Stute (1999); Park and Whang (1999); Whang (2000); Dominguez and Lobato (2003). The recent test of the MDH by Escanciano and Velasco (2007a) is based on the work of Hong (1999a,b) and uses the exponential function as well.

The strong focus on the exponential function as a tool for deriving specification tests for deviations from parametric null models has been questioned in Stinchcombe and White (1998) who argue that there is nothing special about the exponential function (or indeed the indicator function) that makes it capable of detecting arbitrary deviations from parametric null models. They show that most bases of functions are capable of this, with the exception of polynomials. In particular, they note that neural network specifications are powerful approximators whose approximation properties have been established formally in the literature (see, e.g., Hornik, Stinchcombe, and White (1989)).

In specification testing, the focus on the exponential and indicator functions can be partly explained by the lack of robust and efficient estimation algorithms for flexible non-linear specifications that could play the role of  $\mu$ . As a result, focus has been placed on the exponential function, or, more generally, specifications that are restricted to involve linear combinations of basis functions, such as trigonometric functions. Such basis functions do not involve unknown parameters and, therefore, estimation boils down to linear least squares estimation of the linear combination coefficients. Such restrictions, however, have considerable

costs in the sense that many classes of powerful flexible nonlinear specifications are excluded.

In a series of papers, Blake and Kapetanios (2007, 2000, 2003a,b) have introduced a new class of neural networks in the context of a diverse set of testing problems in econometrics. These neural network specifications based on radial basis functions neural networks (RBFNN), provide a novel way for alleviating the aforementioned estimation (and in some cases identification) problem. This work focused on small sample performance but, recently, work by the same authors (Kapetanios and Blake (2007)) have provided a rigorous justification for their specifications using ideas from boosting.<sup>1</sup>

This paper uses the equivalence in (1), to propose regression based tests for the MDH, based on the neural network testing procedures of Blake and Kapetanios (2000, 2003a,b). In particular, (1) immediately implies that for some  $\mu(I_{t-1})$ , a Wald test of the null hypothesis that  $\alpha = 0$  in

$$y_t = \alpha\mu(I_{t-1}) + \epsilon_t$$

where  $\epsilon_t$  is assumed to be a martingale difference process, can be used to construct valid tests for the MDH. Unlike previous work on MDH tests we use neural networks approximations to choose  $\mu$ . We provide novel theoretical results for our testing procedure and carry out a Monte Carlo study which suggests that the new procedures provide superior power performance compared to the most powerful existing MDH tests in the literature.

The structure of the paper is as follows: Section 2 presents the new testing procedures. Section 3 provides some theoretical results for them. Section 4 presents a Monte Carlo study. Section 5 provides an empirical application. Finally, Section 6 concludes.

## 2 Setup

Consider a stochastic process  $y_t, t = 1, \dots, T$ . We wish to test the MDH that

$$E(y_t|I_{t-1}) = \delta \quad \forall t \quad a.s. \tag{2}$$

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<sup>1</sup>Boosting refers to a set of algorithms which have become very popular in disciplines such as machine learning and, more recently, statistics, in the context of classification and prediction (see, e.g., Freund and Schapire (1996), Friedman, Hastie, and Tibshirani (2000), Schapire (2002), Friedman (2001) and Buhlmann (2006)).

By the equivalence stated in (1) it follows that there is no  $\mathcal{F}_{t-1}$  measurable function  $\mu(I_{t-1})$  such that  $\alpha \neq 0$  in a regression model of the form

$$y_t = \alpha' \mu(x_t) + \epsilon_t \quad (3)$$

where  $x_t = (y_{t-1}, \dots, y_{t-q})'$ . Therefore, the problem of testing the MDH becomes one of testing  $\alpha = 0$ , for some appropriate function  $\mu(\cdot)$  where both  $\mu(\cdot)$  and  $\alpha$  can be either scalar, or more generally vectors of, functions and coefficients respectively. A standard Wald test can be used for this test. The main issue is to construct  $\mu(\cdot)$  so as to have appropriate performance both under the null MDH and under a wide variety of alternative hypotheses. We wish to provide a portmanteau test and so need to provide a method that is nonparametric in the sense that it can capture any function for which  $\alpha \neq 0$ .

Given the work of Blake and Kapetanios (2007, 2000, 2003a,b), who show that RBFNN specifications can be used to construct powerful tests for a wide variety of alternative hypotheses in different regression contexts, our aim is to estimate the unknown regression function by an RBFNN series expansion of the form

$$\hat{\mu}(x_t) = \sum_{i=1}^m c_i \psi(x_t, t_i, \sigma_T) \quad (4)$$

where the RBF nodes,  $\psi(x_t, t_i, \sigma_T)$ , are radially symmetrical, integrable, bounded functions and  $t_i$  are referred to as the centres of the RBFs. Examples include the Gaussian function of the form  $\exp\left(-\left(\frac{\|x-t_i\|}{\sigma_T}\right)^2\right)$ , or the multiquadratic function  $\left(1 + \left(\frac{\|x-t_i\|}{\sigma_T}\right)^2\right)^{-1}$ ,  $\sigma_T > 0$ , where  $\|\cdot\|$  denotes Euclidean distance. Obviously, estimation of (4) is challenging since unlike standard series expansions, there are two problems that need attention. The first is that  $\psi(x, t_i, \sigma_T)$  contain unknown parameters, in particular the centres, and the second is that the nodes are not ranked so that the choice of the nodes in the series expansion is not obvious. Once the order of the nodes and the centres are determined the series expansion can be estimated by least squares.

A popular approach to the solution of the above problems was suggested by Orr (1995) who suggested a form of forward selection procedure using every data point as potential centres. In a series of papers, Blake and Kapetanios (2007, 2000, 2003a,b) have modified that algorithm for specifically econometric applications with some success. In this paper we modify it further to bring it more in line with the regression based boosting algorithm of Buhlmann (2006) and the analysis used in Kapetanios and Blake (2007) who provide the

first theoretical results for this algorithm. We define this algorithm as Algorithm 1 below, and label it as the *(RBF) MDH Boosting Algorithm*.

**Algorithm 1** *(RBF) MDH Boosting algorithm*

1. Let  $\sigma_T$  be some sequence such that  $\sigma_T = o(1)$ . We construct the initial set of  $T$  RBF nodes given by:  $\Psi^{(1,\dots,T)} = \{\psi(x, x_1, \sigma_T), \psi(x, x_2, \sigma_T), \dots, \psi(x, x_T, \sigma_T)\}$ .
2. These are ranked according to their ability to reduce the residual variance, when each  $\psi(x_t, x_i, \sigma_T)$ ,  $i = 1, \dots, T$ , is entered individually in (4).
3. The node that minimises the residual variance becomes the first node in the ranking of the nodes. Denote this node by  $\psi(x, x_{\mathcal{S}_1}, \sigma_T)$ . Denote the residual from the regression of  $y_t$  on  $\psi(x_t, x_{\mathcal{S}_1}, \sigma_T)$ , by  $y_t^{(1)}$ . Let  $\tilde{\mathcal{S}}_1 = \{\mathcal{S}_1\}$ . Let  $\Psi^{(1,\dots,T)/\tilde{\mathcal{S}}_1}$  be the set of nodes in  $\Psi^{(1,\dots,T)}$  apart from the nodes indexed by the elements of  $\tilde{\mathcal{S}}_1$ .
4. Set  $i = 1$ .
5. The nodes in  $\Psi^{(1,\dots,T)/\tilde{\mathcal{S}}_1}$  are ranked according to their ability to reduce the residual variance of  $y_t^{(i)}$ , when  $y_t^{(i)}$  is regressed on each  $\psi(x_t, x_i, \sigma_T)$ ,  $i \in \tilde{\mathcal{S}}_1$ .
6. The node that minimises the residual variance becomes the  $i+1$ -th node in the ranking of the nodes. Denote this node by  $\psi(x, x_{\mathcal{S}_{i+1}}, \sigma_T)$ . Denote the residual from the regression of  $y_t^{(i)}$  on  $\psi(x_t, x_{\mathcal{S}_{i+1}}, \sigma_T)$ , by  $y_t^{(i+1)}$ . Let  $\tilde{\mathcal{S}}_{i+1} = \tilde{\mathcal{S}}_1 \cup \{\mathcal{S}_{i+1}\}$ . Let  $\Psi^{(1,\dots,T)/\tilde{\mathcal{S}}_{i+1}}$  be the set of nodes in  $\Psi^{(1,\dots,T)}$  apart from the nodes indexed by the elements of  $\tilde{\mathcal{S}}_{i+1}$ .
7. If  $i = m$  for some  $m = m_T \rightarrow \infty$  stop, else set  $i = i + 1$  and go to Step 5.

Some remarks are in order for this algorithm.

**Remark 1** *The choice for  $m$  is not discussed in Algorithm 1 apart from noting that  $m \rightarrow \infty$ . Theorem 1 of Kapetanios and Blake (2007) suggests that the maximum possible rate is logarithmic in  $T$ .*

**Remark 2** *The sequence  $\sigma_T$  is left unspecified in Algorithm 1. The proof of Theorem 1 of Kapetanios and Blake (2007) suggests that the choice  $\sigma_T = O((\ln \ln T)^{-1})$  is acceptable. Given the very slow rate involved, it is reasonable to consider ad hoc data-based values following the practice established by Orr (1995). Accordingly, in practice this tuning parameter is set such that  $\sigma_T = \sigma$  where  $\sigma = 2 \max_t |x_t - x_{t-1}|$ . This is our choice for the Monte Carlo study.*

**Remark 3** *The choice of the initial set of RBF nodes given by:*

$$\Psi^{(1,\dots,T)} = \{\psi(x, x_1, \sigma_T), \psi(x, x_2, \sigma_T), \dots, \psi(x, x_T, \sigma_T)\}$$

*may be straightforwardly generalised to  $\Psi^{(1,\dots,p_T)}$  where  $p_T$  is chosen to reflect a subset of the observations or possibly be of a larger order than  $T$ .*

**Remark 4** *Algorithm 1 is more computationally demanding than that used in Blake and Kapetanios (2007, 2000, 2003a,b). There the nodes are ranked only once according to their ability to reduce the residual variance, when entered individually in (4). The two algorithms are very similar. The cost is an increase in computational effort of the order of  $T(T+1)/2$  for Algorithm 1.*

**Remark 5** *Although the discussion in this paper is couched in terms of RBFNNs it is worth noting that extensions to other neural network specifications such as neural networks based on logistic function nodes are possible once a grid of possible parameter values is constructed. One such specification is considered in White (2006) where an algorithm is constructed but no formal theoretical justification for it is given. The advantage of RBFNNs, in the context of Algorithm 1, is the fact that the construction of the grid is obtained by using the actual sample observations thus ensuring an appropriate coverage of the relevant state space for the processes under consideration.*

Once an ordered set of  $m$  RBFNN nodes is available via algorithm 1, a data dependent method can be used to determine the final number of nodes to enter in the testing regression. Guay and Guerre (2006) provide a theoretical analysis of tests based on similar series expansions<sup>2</sup> and suggest the use of a data dependent method to determine the final number of nodes to enter in the testing regression. Their method depends on a penalty term of order  $(\ln \ln T)^{1/2}$  to counterbalance the increase in fit from the use of more nodes in the testing regression. In particular, they suggest that the number of nodes,  $k^*$ , finally used in the testing regression be given by

$$k^* = \operatorname{argmax}_{k=1,\dots,m} \{R_k - \zeta_{T,k}\}$$

where  $\zeta_{T,k} = k - \sqrt{2\gamma_T k}$  is a penalty term,  $\gamma_T = \ln \ln T$ ,  $R_k = y' \Psi_k (\Psi_k' \Psi_k)^{-1} \Psi_k' y$ ,  $\Psi_k = (\psi_{1,k}, \dots, \psi_{k,k})$  and  $\psi_{i,k} = (\psi(x_1, x_{\mathcal{S}_i}, \sigma_T), \dots, \psi(x_T, x_{\mathcal{S}_i}, \sigma_T))'$ ,  $i = 1, \dots, k$ . This is similar to the method adopted in Blake and Kapetanios (2007, 2000, 2003a,b). The penalty terms used in Blake and Kapetanios (2003b) are the ones associated with either the Akaike or the Bayesian

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<sup>2</sup>Guay and Guerre (2006) advocate a trigonometric expansion.

information criteria. These penalties are not optimal in the sense of Guay and Guerre (2006) since the Akaike penalty term, given by a finite constant, results in a test which does not have an asymptotic  $\chi^2$  approximation whereas the Bayesian criterion, with a penalty term of order  $\ln T$ , is too parsimonious. In the context of the information criterion-based work of Blake and Kapetanios (2003b) the Hannan-Quinn criterion with a penalty term of order  $\ln \ln T$  seems a more appropriate choice. All of these choices are explored in the Monte Carlo study. Finally, the joint significance of the coefficients of the chosen set of nodes and  $\beta$ , are tested via a Wald test in the following regression

$$y_t = \beta' x_t \sum_{i=1}^{k^*} \alpha_i \psi(x_t, x_{S_i}, \sigma_T) + \epsilon_t \quad (5)$$

We refer to this test as the RBFNN-BOOST test.

**Remark 6** *In the above discussion we have not addressed the issue of choosing the number of lags of the process  $y_t$  to be used in the construction of the neural network nodes. Although the above analysis, and the theory of the next section, assumes a fixed  $q$ , it is straightforward to envisage the possibility of choosing  $q$  via a criterion such as any of those discussed above. In this more general case, the analysis would consider two iterations to fully construct the final regression model in (5). Firstly one would consider all possible values for  $q = 1, \dots, q_{max}$ , and for each  $q$  a value of  $k^*$  would be chosen. Then, the criterion would be used to jointly select a  $(q, k^*)$  pair over all permutations.*

### 3 Theoretical Results

In this section we present the main theoretical results for the RBFNN test. The following assumptions will be needed.

**Assumption 1**  $E|\epsilon_t^s| < \infty$  for some  $s > 8$

**Assumption 2** Under the alternative hypothesis,  $\mu(\cdot)$  is  $L_2$ -bounded.

**Assumption 3** Under MDH, the sequence  $\{\epsilon_t\}_{t=-\infty}^{\infty}$  is a martingale difference sequence with  $E(\epsilon_t|\mathcal{F}_t) = 0$ ,  $E(\epsilon_t^2|\mathcal{F}_t) = \sigma^2$ .

**Assumption 4**  $\epsilon_t$  is a stationary  $\alpha$ -mixing processes with  $\alpha$ -mixing coefficients given by  $\alpha(k) = C_1 k^{-C_2}$ ,  $C_1 > 0$ ,  $C_2 > 1$ .  $p_T = o(T^{1/4})$ .

**Assumption 5**  $y_t$  has a density  $f(\cdot)$  which is bounded away from zero and infinity.



**Remark 7** *Assumption 4 provides dependence structures for  $\epsilon_t$ . It further sets a rate for  $p_T$  related to these dependence assumptions. Note that there is a trade-off between the dependence structure of  $\epsilon_t$  and the rate allowed for  $p_T$  for the approximation properties of Algorithm 1, which is discussed in Kapetanios and Blake (2007), but not explored here since a mixing assumption is required for the results of Guay and Guerre (2006). Further, note that the constant variance condition in Assumption 3 may be relaxed to  $E(\epsilon_t^2|\mathcal{F}_t) = \sigma^2(I_{t-1})$  where  $\sigma(\cdot)$  is continuous and bounded away from zero. We choose a simpler structure for the sake of clarity.*

Then, the following theorems proved in the appendix hold:

**Theorem 1** *Let assumptions 1-5 hold. Then, under the MDH, the RBFNN-BOOST test based on the Wald test for the null hypothesis that  $\alpha_1 = \alpha_2 = \dots = 0$  in (5) is asymptotically of level  $\alpha$  when  $k^*$  is chosen by maximising  $R_k - \zeta_{T,k}$  over  $k$  and the penalty term,  $\zeta_{T,k}$ , is either that of Guay and Guerre (2006) or that associated with either the Bayesian or the Hannan-Quinn information criterion.*

**Theorem 2** *Let assumptions 1-5 hold. Consider the sequence of alternatives  $\Delta_T$  in  $H_1(C\rho_T, L, s)$  where*

$$H_1(C\rho_T, L, s) = \left\{ \Delta_T \in \mathcal{C}(L, s), E(\Delta_T(x_t)I(x_t \in \Lambda))^2 \geq \rho_T \right\}, \quad (6)$$

$$\mathcal{C}(L, s) \left\{ \Delta(\cdot) : \sup_{x, x' \in \Lambda} \frac{|\Delta(x) - \Delta(x')|}{\|x - x'\|} \leq L \right\},$$

for some  $\Lambda \in \mathbb{R}^q$ ,  $\rho_T = \log_a T$ , where  $a$  is defined in Theorem 4 and for some unknown finite  $L$  and some unknown finite and sufficiently large  $s$ . Then, if  $\zeta_{T,k} = k - \sqrt{2\gamma_T k}$  and  $\gamma_T = O(\ln \ln T)$ , the RBFNN-BOOST test based on the Wald test of the null hypothesis that  $\alpha_1 = \alpha_2 = \dots = 0$  in (5), is consistent.

**Theorem 3** *Let assumptions 1-5 hold. Consider the sequence of alternatives  $r_T \Delta_T$  in  $\mathcal{C}(L, s)$  for some unknown  $L$  and some unknown and sufficiently large  $s$ , where*

$$E(\Delta_T(x_t)I(x_t \in \Lambda))^2 \geq 1,$$

$$\sup_{x \in \Lambda} = O(1),$$

for some  $\Lambda \in \mathbb{R}^q$ . Then, if  $\zeta_{T,k} = k - \sqrt{2\gamma_T k}$  and  $\gamma_T = O(\ln \ln T)$  the the RBFNN-BOOST test based on the Wald test of the null hypothesis that  $\alpha_1 = \alpha_2 = \dots = 0$  in (5) is consistent, provided that  $r_T = o(T^{-1/2})$ .

**Remark 8** *The core of the proofs of all three theorems above is Theorem 1 of Kapetanios and Blake (2007), reproduced for convenience as Theorem 4 in the appendix which provides a theoretical result on the approximation properties of Algorithm 1. The rate of convergence to the true unknown regression function  $\mu$ , under the alternative, given in Theorem 4, is rather sharp. Not all logarithmic rates are accommodated.*

**Remark 9** *Theorem 2 relates to alternatives with varying smoothness characteristics as evidenced by the family of functions in (6). Note that for this family of functions the MHD Boosting algorithm can only detect alternatives that tend to zero at a logarithmic rate unlike trigonometric approximations which can detect alternatives that tend to zero at a polynomial rate. However, it is worth noting two things: firstly, the small sample performance of the RBFNN test discussed in the next section, suggests that the ability of RBFNN specifications to adapt in a data dependent fashion, not only in terms of the number of nodes, but also in terms of the shape of nodes gives it a distinct advantage in terms of power performance. Secondly, the logarithmic rate is only the consequence of the use of boosting. RBFNN specifications have polynomial approximation rates and if nonlinear estimation of the RBFNN specification was practical a polynomial rate would be obtained. Theorem 3 relates to smooth alternative hypotheses. In this case the RBFNN test can achieve a detection rate arbitrarily close to the parametric one.*

## 4 Monte Carlo Study

Having provided a thorough analysis of the theoretical properties of the newly proposed MDH tests, we provide a Monte Carlo study of their small sample properties in this section. Comparability with results of Monte Carlo studies of other MDH tests is very important. Therefore, we follow very closely (and in the case of power experiments exactly) the Monte Carlo study of Escanciano and Velasco (2007a). As discussed in the previous section, the RBFNN-BOOST test is similar but more computationally intensive than the tests proposed in Blake and Kapetanios (2000, 2003a,b). Further, RBFNN-BOOST is likely to be more powerful than those tests. Since we feel that the premium on computational ease is considerable and since the test of Blake and Kapetanios (2003a) will provide a lower bound in terms of power properties for the power of RBFNN-BOOST we choose to also use the RBFNN specification of Blake and Kapetanios (2003a) in our Monte Carlo study. In particular, we use the following algorithm for the RBFNN test.

**Algorithm 2** *(RBF) MDH algorithm*

1. We construct  $T$  initial RBF terms given by  $\Psi^{(1,\dots,T)}$  where  $\Psi^{(1,\dots,T)}$  is defined in Algorithm 1.
2. These are ranked according to their ability to reduce the residual variance, when entered individually in (5) (i.e. when  $x_t$  and only one nonlinear regressor is included in (5)).
3. Penalised likelihood criteria are used to determine how many of the  $T$  sorted RBF terms will eventually enter (5).

We then test for the significance of the included hidden units together with the linear part of the specification using a Wald test. We refer to this as the RBFNN test. The question of which penalised likelihood criteria to use is very important. In particular, the choice of the penalty term is very important. The theory of the preceding section has used the criterion suggested by Guay and Guerre (2006), and denoted for our purposes as GG, which, as noted before, is of the form  $k + \sqrt{2 \ln \ln Tk}$  and has some theoretical optimality property. However, this property is not that relevant for this analysis since the order of the approximation has to be logarithmic for the boosting algorithm to be operational as discussed in the previous section.

On the other hand, Blake and Kapetanios (2000, 2003a,b) have used standard information criteria which are of the form  $\gamma_T k$  where  $\gamma_T$  takes the value  $2, \ln T$  and  $2 \ln \ln T$  for the Akaike (AIC), Bayesian (BIC) and Hannan-Quinn (HQ) information criteria respectively. The AIC is theoretically inappropriate since its associated penalty term is too small and so the resulting test does not have an asymptotic  $\chi^2$  distribution. It is straightforward to see that the other two criteria have an asymptotic  $\chi^2$  distribution, as shown in Theorem 1. In fact, their penalty terms are too parsimonious with the one associated with the BIC being the most parsimonious. However, the work of Blake and Kapetanios (2000, 2003a,b) suggests that the RFB approximations are efficient and so few approximation terms are sufficient for obtaining powerful tests. Hence, the BIC was found to be best since it minimised overrejection under the null to perfectly acceptable levels. To investigate all these issues we consider all four penalty terms (AIC, BIC, HQ and GG) in our Monte Carlo study.

We also wish to consider tests based on polynomial approximations and on logistic neural networks along the lines of Teräsvirta, Lin, and Granger (1993) and Lee, White, and Granger (1993). Both approximation classes were considered by Blake and Kapetanios (2000, 2003a,b) as well and were found to be reasonable alternatives to RBFNN approximations. In the case

of the polynomial approximations we use again a data dependent method for determining the order of the approximation using the penalised likelihood criteria discussed above. Approximations of orders 2, 3 and 4 are considered. For the logistic neural network approximations we use the approach of Lee, White, and Granger (1993) and simply augment the set of regressors whose significance we test with the linear part of the regression. Thus, this test becomes one for the MDH rather than of neglected nonlinearity as in Lee, White, and Granger (1993). The approach of Lee, White, and Granger (1993) suggests that the specification of each neural network node is given by  $\phi(\gamma'x_t)$  where  $\phi(\lambda)$  is the logistic function  $\{1 + \exp(-\lambda)\}^{-1}$ . This is a monotonic function, with output bounded between 0 and 1. The coefficients  $\gamma_j$  are randomly generated from a uniform distribution over  $(\gamma_l, \gamma_h)$ . For given  $k^*$ , the constructed regressors  $\phi(\gamma_j'x_t)$ ,  $j = 1, \dots, k^*$ , may suffer from multicollinearity. Lee, White, and Granger (1993) suggest that  $\tilde{k}^*$  largest principle components of the constructed regressors excluding the largest one be used as extra regressors. We set  $\gamma_l = -2$ ,  $\gamma_h = 2$ ,  $k^* = 10$  and  $\tilde{k}^* = 2$  as in the original paper. Recent work by White (2006) suggests that a similar boosting approach can be used with the logistic neural network specifications. However, we note here that the construction of the parameter grid is much less intuitive than that for the RBFNN. The tests based on the approaches of Teräsvirta, Lin, and Granger (1993) and Lee, White, and Granger (1993) are denoted by TLG and LWG respectively.

## 4.1 Experiment design

Following the Monte Carlo study of Escanciano and Velasco (2007a) we consider the following experiments. The first three experiments involve processes that satisfy the MDH and therefore provide information about the size properties of the tests, whereas the rest of the experiments involve processes that are not martingale difference sequences and therefore provide information about the power properties of the tests. The power experiments are exactly the same as in Escanciano and Velasco (2007a) so as to enable valid comparisons with their power results. We have not considered the long memory power experiment of Escanciano and Velasco (2007a) since it is an experiment involving a linear model, and by the AR representation of long memory processes (see, e.g., Beran (1994) and Poskitt (2005)) it is obvious that our methods will work extremely well. The form of the second and third size experiments have been retained. Parameter values were rounded but remain close to those used in Escanciano and Velasco (2007a) (e.g., we use 0.9 rather than 0.936 for the autoregressive parameter of the stochastic volatility model). Throughout the experiment description,  $\epsilon_t, u_t \sim NID(0, 1)$ .

1.  $y_t = \epsilon_t$ , (IID)
2.  $y_t = \epsilon_t \sigma_t$ , with  $\sigma_t^2 = 0.001 + 0.01y_{t-1}^2 + 0.9\sigma_{t-1}^2$ , (GARCH)
3.  $y_t = \epsilon_t \exp(\sigma_t)$ , with  $\sigma_t = 0.9\sigma_{t-1} + 0.05u_t$ , (SV)
4.  $y_t = \epsilon_{t-1}\epsilon_{t-2}(\epsilon_{t-2} + \epsilon_t + 1)$ , (NLMA)
5.  $y_t = \epsilon_t + 0.15\epsilon_{t-1}y_{t-1} + 0.05\epsilon_{t-1}y_{t-2}$ , (BILIN1)
6.  $y_t = \epsilon_t + 0.25\epsilon_{t-1}y_{t-1} + 0.15\epsilon_{t-1}y_{t-2}$ , (BILIN2)
7.  $y_t = \epsilon_t + x_t - x_{t-1}$ ,  $x_t = 0.85x_{t-1} + u_t$ ,  $\epsilon_t$ , (NDAR)
8.  $y_t = -0.5y_{t-1}I(y_{t-1} \geq 1) + 0.4y_{t-1}I(y_{t-1} < 1) + \epsilon_t$ , (SETAR)
9.  $y_t = 0.6y_{t-1}\exp(-0.5y_{t-1}^2) + \epsilon_t$ , (EXP)

All tests use the first lag of the process to construct the RBFNN nodes. The first lag is also used for the TLG and LWG tests. Rejection probabilities for a nominal significance level of 95%, produced using 1000 replications, are reported in Tables 1-4.

## 4.2 Size results

Looking at the performance of the RBFNN and RBFNN-BOOST tests under the null hypothesis, it is clear that depending on the penalty term used there is some overrejection. This is most severe for the AIC followed by GG and HQ, as expected given their relative parsimony. The BIC performs best in this respect with no noticeable overrejection. Similar patterns occur for the TLG and LWG tests. The results accord with our experience in related applications, where searching for any form of significant neglected structure tends to induce overrejection. The good performance of the BIC in this respect removes any need to resort to bootstrap size correction.

## 4.3 Power results

An analogous pattern emerges for the power experiments. The tests based on BIC have slightly lower power. Since these are the only tests that do not overreject and the loss of power compared to the other tests is minimal, these tests seem preferable. In terms of relative power performance, tests have more power against the SETAR alternative, followed by the BILIN1, NLMA, EXP, BILIN2 and NDAR alternatives. The NDAR alternative seems to be extremely difficult to detect. This is corroborated by the Monte Carlo results of Escanciano

and Velasco (2007a). The RBFNN test seems to be more powerful than the TLG and LWG tests for all experiments considered. The RBFNN-BOOST test appears only slightly more powerful than the RBFNN test and that superiority only appears in some experiments. We feel that use of the RBFNN-BOOST may not be necessary given the extra computational cost.

#### 4.4 Comparison with Escanciano and Velasco (2007a,b)

We chose our experiments to facilitate comparison of the rejection probabilities under the alternative of our new tests with those proposed in Escanciano and Velasco (2007a). Since we use exactly the same experimental design a comparison is possible without replicating their results. Note that the bootstrap  $\mathbf{D}_n^2$  test of Escanciano and Velasco (2007a) has more power than all the alternative tests considered in that paper. The new tests we propose are uniformly more powerful than that test with a single exception.

The one exception relates to experiment EXP. However, we shouldn't feel too bad about that as it is only to be expected. The data generation process for that experiment has an exponential structure. The  $\mathbf{D}_n^2$  uses the exponential function and considers the covariance between  $y_t$  and  $\exp(y_{t-j})$ ,  $j > 0$ . Hence it is essentially a parametric test for this sort of deviation from the MDH. Overall, it is pretty clear that the newly proposed tests have superior small sample power performance to  $\mathbf{D}_n^2$  and (by the Monte Carlo results of Escanciano and Velasco (2007a)) to many other MDH tests.

Finally, note that three of the experiments of Escanciano and Velasco (2007a) we consider (NLMA, BILIN1, BILIN2), are also used by Escanciano and Velasco (2007b) for an alternative MDH test which is similar to that of Escanciano and Velasco (2007a) but uses the indicator function rather than the exponential one. From the Monte Carlo study results of Escanciano and Velasco (2007b) we see that the RBFNN test has superior power properties compared to the test of Escanciano and Velasco (2007b).

#### 4.5 The benefits of the RBF approach

As a final comment it is worth noting that in small samples the nature and shape of the nodes that form a series approximation are more important than the number of nodes. We offer two pieces of evidence to support this. First, we note that the power performance depends only slightly on the choice of the penalty terms in the penalised likelihood criteria.

Adding additional nodes does not improve the performance and a more parsimonious test can safely be used. Second, as a check we considered (but have not reported) a trigonometric approximation of the form considered in Guay and Guerre (2006). However, the power performance of the tests based on this approximation was significantly inferior to that of the RBFNN based test.

## 5 Empirical Application to Stock Returns

In this section, we provide an empirical application that illustrates the potential of the new test to evaluate the martingale difference hypothesis. It is widely thought that stock returns are ‘close’ to unpredictable, and empirical analysis to predict future returns based on past return data has little or no explanatory power (see, for example, Cochrane, 2005, Chapter 20). We apply our test to stock returns to see if this can be supported. As it is sometimes difficult to draw meaningful conclusions from the empirical analysis of a single series for the performance of a new statistical test, we consider the large S&P 500 dataset which allows us to draw wider conclusions based on the proportion of the series which reject.

Weekly returns data were obtained from Datastream, spanning the period 01/01/1993-20/01/2004 and comprising 575 weekly observations. We consider only companies for which data are available throughout the period, a total of 412 series. We normalise the returns series to have mean equal to zero and variance equal to one prior to testing. We report the probability values for the test of the martingale difference hypothesis, carried out on the 412 company return series in Tables 5-8. Probability values below 0.05, and the company names to which they correspond, are reported in bold typescript for easy identification.

As we can see for these Tables a large minority of the series (165 stock returns in total as we can see from the Tables) are in fact found to reject the martingale difference null hypothesis at the 95% significance level. This is almost exactly 40% of the series tested, far higher than what we would expect to occur if returns were indeed unpredictable. Whilst the use of macroeconomic factors can improve forecastability (see, for example, Lettau and Ludvigson, 2001) our analysis suggests that more general nonlinear specifications also may have more forecasting power than traditional linear specifications.

## 6 Conclusions

The martingale difference restriction is an outcome of many theoretical analyses in economics and finance. A large body of econometric literature deals with tests of that restriction. We provide new tests based on radial basis function neural networks. Our work is based on the test design of Blake and Kapetanios (2000, 2003a,b). However, unlike that work we can provide a formal theoretical justification for the validity of these tests using approximation results from Kapetanios and Blake (2007). These results take advantage of the link between the algorithms of Blake and Kapetanios (2000, 2003a,b) and boosting. We carry out a Monte Carlo study of the properties of the new tests and find that they have superior power performance to all existing tests of the martingale difference hypothesis we consider. An empirical application to the S&P500 constituents illustrates further the usefulness of our new tests.



# Appendix

The proof of Theorems 1-3 consist of showing that all conditions used in Theorems 1-3 of Guay and Guerre (2006) and therefore by extension, in the relevant parts of Propositions 1, 2 and Lemmas 1, A.1-A3 of the same paper, for the trigonometric series expansion, hold for the neural network expansion apart from the different polynomial approximation rate. These conditions, and the location of their use in the context of Guay and Guerre (2006), in parentheses, are (A1) uniform boundedness and orthonormality of the basis functions used to construct the approximation to the unknown regression function, (Lemmas A.1-A.3); (A2) The cardinality of the set of the possible number of nodes for the approximation should be  $\ln T$ , (Lemma A.2); (A3) The series expansion approximates the unknown regression function at a polynomial rate (Lemma 1). (A2) and (A3) follow immediately from Theorem 1 of Kapetanios and Blake (2007) and algorithm 1. We reproduce Theorem 1 of Kapetanios and Blake (2007) for convenience.

**Theorem 4 (Theorem 1 of Kapetanios and Blake (2007))** *Let assumptions 1-5 hold. The estimate of the regression function  $\mu(x_t)$ , obtained using the iterative boosting algorithm 1 and denoted  $\hat{\mu}(x_t)$ , satisfies  $\hat{\mu}(x_t) - \mu(x_t) = o_p(m^{-1/C_1})$ , for all  $C_1 > 6$  and some sequence  $\sigma_T = o(1)$ , if  $m < \log_a T$ , for all  $a$  that satisfy  $\log_a e < \frac{\ln(5/2)}{4}$ . As a by-product of this estimation, an ordering of the radial basis function neural network nodes is obtained.*

We investigate (A1). The set of radial basis functions is uniformly bounded by definition for any radial basis function. However, the ordered set of functions arising out of the boosting algorithm is not orthonormal. Nevertheless, it can be made orthonormal using a number of possible orthonormalisation algorithms. We consider the Gram-Schmidt orthonormalisation algorithm. Let  $\Psi_m = \{\psi(x, t_1, \sigma_T), \dots, \psi(x, t_m, \sigma_T)\}$  denote a set of radial basis functions used, in a regression, to approximate  $\mu_1$ . Let the transformed set of functions be denoted  $\check{\Psi}_m = \{\check{\psi}(x, t_1, \sigma_T), \dots, \check{\psi}(x, t_m, \sigma_T)\}$  where  $\check{\Psi}_m$  has been obtained from  $\Psi_m$  by Gram-Schmidt orthonormalisation as follows:

$$\check{\psi}(x, t_1, \sigma_T) = \frac{\psi(x, t_1, \sigma_T)}{\|\psi(x, t_1, \sigma_T)\|} \quad (7)$$

$$\check{\psi}(x, t_2, \sigma_T) = \frac{\psi(x, t_2, \sigma_T) - \left\langle \psi(x, t_2, \sigma_T), \check{\psi}(x, t_1, \sigma_T) \right\rangle \check{\psi}(x, t_1, \sigma_T)}{\left\| \psi(x, t_2, \sigma_T) - \left\langle \psi(x, t_2, \sigma_T), \check{\psi}(x, t_1, \sigma_T) \right\rangle \check{\psi}(x, t_1, \sigma_T) \right\|} \quad (8)$$

...

$$\check{\psi}(x, t_m, \sigma_T) = \frac{\psi(x, t_m, \sigma_T) - \sum_{i=1}^{m-1} \langle \psi(x, t_m, \sigma_T), \check{\psi}(x, t_i, \sigma_T) \rangle \check{\psi}(x, t_i, \sigma_T)}{\left\| \psi(x, t_m, \sigma_T) - \sum_{i=1}^{m-1} \langle \psi(x, t_m, \sigma_T), \check{\psi}(x, t_i, \sigma_T) \rangle \check{\psi}(x, t_i, \sigma_T) \right\|} \quad (9)$$

In order to prove the equivalence of using either  $\Psi_m$  or  $\check{\Psi}_m$  in a regression to approximate  $\mu_1$  we simply note that for all  $i$

$$\check{\psi}(x, t_i, \sigma_T) = \sum_{j=1}^i \check{c}_{ji} \psi(x, t_j, \sigma_T)$$

where the  $\check{c}_{ji}$ 's are determined in the recursions (7)-(9). Therefore,

$$\begin{aligned} \psi(x; m) &= \sum_{i=1}^m \check{c}_i \check{\psi}(x, t_i, \sigma_T) = \sum_{i=1}^m \check{c}_i \left( \sum_{j=1}^i \check{c}_{ji} \psi(x, t_j, \sigma_T) \right) = \\ &= \sum_{i=1}^m \sum_{j=1}^i \check{c}_i \check{c}_{ji} \psi(x, t_j, \sigma_T) = \sum_{i=1}^m c_i \psi(x, t_i, \sigma_T) \end{aligned}$$

where by grouping appropriate terms

$$c_i = \sum_{\ell=i}^m \check{c}_\ell \check{c}_{\ell i}$$

To complete the proof of Theorems 1-3 we need to establish two more facts. The first relates to the validity of using the penalty terms associated with the Bayesian and Hannan-Quinn information criteria for Theorem 1. But given that these penalty terms are of a higher order than  $(\ln \ln T)^{1/2}$  the result follows immediately. The second fact relates to the relaxation of the assumption that the minimum possible order  $k_{min}$  over which to search for  $k^*$  has to tend to infinity, that was made in Guay and Guerre (2006). That assumption is made in Guay and Guerre (2006) since they consider the case where a preliminary estimation leads to a set of residuals which are then tested for lack of structure (in our case the MDH hypothesis). The assumption is needed to make the estimation error of the preliminary estimation negligible. Since we do not consider any preliminary estimation this assumption is not needed. This completes the proof. ■

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Table 1: Results for RBFNN-BOOST Test								
	AIC				BIC			
Exp/T	50	100	200	300	50	100	200	300
IID	0.113	0.099	0.070	0.088	0.089	0.069	0.050	0.059
GARCH	0.158	0.107	0.105	0.082	0.104	0.074	0.065	0.058
SV	0.118	0.103	0.104	0.100	0.086	0.068	0.065	0.063
BILIN1	0.329	0.514	0.797	0.912	0.278	0.464	0.777	0.898
BILIN2	0.604	0.858	0.991	0.998	0.559	0.838	0.988	0.998
NDAR	0.157	0.115	0.132	0.125	0.121	0.097	0.090	0.100
NLMA	0.539	0.644	0.718	0.783	0.490	0.553	0.613	0.668
SETAR	0.608	0.880	0.990	0.999	0.555	0.847	0.987	0.998
EXP	0.387	0.618	0.895	0.973	0.332	0.537	0.833	0.934
	HQ				GG			
Exp/T	50	100	200	300	50	100	200	300
IID	0.105	0.084	0.064	0.075	0.112	0.097	0.070	0.085
GARCH	0.140	0.094	0.093	0.070	0.155	0.106	0.105	0.080
SV	0.107	0.091	0.091	0.087	0.118	0.100	0.103	0.098
BILIN1	0.308	0.493	0.787	0.904	0.325	0.510	0.796	0.911
BILIN2	0.587	0.843	0.990	0.998	0.604	0.854	0.990	0.998
NDAR	0.144	0.108	0.117	0.114	0.153	0.114	0.131	0.125
NLMA	0.523	0.607	0.683	0.745	0.535	0.640	0.716	0.782
SETAR	0.592	0.866	0.990	0.998	0.606	0.876	0.990	0.999
EXP	0.367	0.578	0.870	0.955	0.382	0.613	0.890	0.970

Table 2: Results for RBFNN Test								
	AIC				BIC			
Exp/T	50	100	200	300	50	100	200	300
IID	0.086	0.088	0.075	0.074	0.069	0.060	0.052	0.055
GARCH	0.126	0.103	0.094	0.075	0.100	0.078	0.068	0.057
SV	0.110	0.095	0.083	0.070	0.090	0.072	0.066	0.051
BILIN1	0.288	0.493	0.768	0.904	0.251	0.462	0.743	0.897
BILIN2	0.538	0.866	0.997	0.999	0.509	0.853	0.993	0.999
NDAR	0.115	0.115	0.114	0.136	0.096	0.086	0.087	0.109
NLMA	0.542	0.604	0.712	0.764	0.484	0.536	0.640	0.678
SETAR	0.570	0.826	0.991	0.999	0.518	0.779	0.973	0.997
EXP	0.370	0.580	0.877	0.978	0.336	0.506	0.831	0.954
	HQ				GG			
Exp/T	50	100	200	300	50	100	200	300
IID	0.081	0.076	0.067	0.067	0.085	0.086	0.074	0.072
GARCH	0.119	0.094	0.083	0.068	0.125	0.102	0.092	0.075
SV	0.104	0.083	0.076	0.066	0.110	0.092	0.081	0.069
BILIN1	0.274	0.479	0.755	0.898	0.284	0.492	0.764	0.901
BILIN2	0.532	0.858	0.995	0.999	0.535	0.865	0.997	0.999
NDAR	0.109	0.110	0.104	0.118	0.114	0.113	0.113	0.132
NLMA	0.523	0.586	0.685	0.736	0.540	0.600	0.707	0.761
SETAR	0.555	0.815	0.984	0.999	0.571	0.823	0.988	0.999
EXP	0.357	0.556	0.865	0.973	0.364	0.570	0.874	0.977

Table 3: Results for TLG Test								
	AIC				BIC			
Exp/T	50	100	200	300	50	100	200	300
IID	0.051	0.068	0.067	0.067	0.050	0.054	0.050	0.052
GARCH	0.075	0.080	0.078	0.072	0.070	0.068	0.060	0.054
SV	0.063	0.073	0.073	0.068	0.061	0.061	0.060	0.051
BILIN1	0.228	0.452	0.755	0.902	0.210	0.430	0.733	0.896
BILIN2	0.463	0.843	0.995	0.999	0.452	0.830	0.991	0.999
NDAR	0.076	0.088	0.101	0.122	0.070	0.074	0.082	0.097
NLMA	0.442	0.544	0.675	0.737	0.417	0.501	0.629	0.657
SETAR	0.492	0.801	0.988	0.999	0.460	0.763	0.981	0.998
EXP	0.287	0.535	0.857	0.973	0.270	0.474	0.808	0.950
	HQ				GG			
Exp/T	50	100	200	300	50	100	200	300
IID	0.051	0.064	0.064	0.061	0.051	0.068	0.067	0.067
GARCH	0.073	0.076	0.071	0.067	0.075	0.080	0.078	0.072
SV	0.062	0.066	0.073	0.064	0.063	0.074	0.073	0.068
BILIN1	0.222	0.441	0.745	0.899	0.228	0.452	0.755	0.902
BILIN2	0.461	0.839	0.994	0.999	0.463	0.843	0.995	0.999
NDAR	0.076	0.085	0.095	0.111	0.075	0.088	0.101	0.122
NLMA	0.439	0.531	0.663	0.716	0.442	0.544	0.675	0.737
SETAR	0.488	0.792	0.987	0.999	0.492	0.801	0.988	0.999
EXP	0.283	0.524	0.846	0.966	0.287	0.535	0.857	0.973

Table 4: Results for LWG Test				
Exp/T	50	100	200	300
IID	0.081	0.092	0.094	0.090
GARCH	0.107	0.113	0.107	0.098
SV	0.092	0.102	0.110	0.096
BILIN1	0.271	0.515	0.786	0.921
BILIN2	0.505	0.866	0.997	0.999
NDAR	0.103	0.117	0.134	0.166
NLMA	0.448	0.532	0.665	0.712
SETAR	0.555	0.845	0.996	1.000
EXP	0.342	0.613	0.897	0.984



Table 5: Probability Values for S&P 500 Series (ABBOTT LABS.- COMPUTER SCIS.)			
Company Name	P. Value	Company Name	P. Value
ABBOTT LABS.	0.079	ADC TELECOM.	0.232
ADOBE SYS.	0.650	ADVD.MICRO DEVC.	0.535
<b>AES (1)</b>	<b>0.000</b>	<b>AFLAC (2)</b>	<b>0.000</b>
AIR PRDS.& CHEMS.	0.228	ALBERTO CULVER 'B'	0.495
<b>ALBERTSONS (3)</b>	<b>0.044</b>	ALCOA	0.760
<b>ALLEGHENY EN. (4)</b>	<b>0.000</b>	<b>ALLEGHENY TECHS. (5)</b>	<b>0.008</b>
ALLERGAN	0.214	<b>ALLIED WASTE INDS. (6)</b>	<b>0.030</b>
ALLTEL	0.277	<b>ALTERA (7)</b>	<b>0.027</b>
ALTRIA GP.	0.469	<b>AMBAC FINANCIAL (8)</b>	<b>0.003</b>
AMERADA HESS	0.777	AMER.ELEC.PWR.	0.594
<b>AMERICAN EXPRESS (9)</b>	<b>0.000</b>	AMER.GREETINGS 'A'	0.308
<b>AMERICAN INTL.GP. (10)</b>	<b>0.000</b>	AMER.POWER CONV.	0.199
AMGEN	0.688	AMSOUTH BANC.	0.070
ANADARKO PETROLEUM	0.234	<b>ANALOG DEVICES (11)</b>	<b>0.023</b>
ANDREW	0.227	<b>ANHEUSER - BUSCH COS. (12)</b>	<b>0.000</b>
<b>AON (13)</b>	<b>0.010</b>	<b>APACHE (14)</b>	<b>0.017</b>
<b>APPLE COMPUTERS (15)</b>	<b>0.032</b>	<b>APPLERA APPD.BIOS. (16)</b>	<b>0.000</b>
APPLIED MATS.	0.255	ARCHER - DANLS.	0.068
ASHLAND	0.358	AT & T	0.799
AUTODESK	0.655	<b>AUTOMATIC DATA PROC. (17)</b>	<b>0.000</b>
<b>AUTONATION (18)</b>	<b>0.001</b>	AUTOZONE	0.522
<b>AVERY DENNISON (19)</b>	<b>0.005</b>	AVON PRODUCTS	0.698
<b>BAKER HUGHES (20)</b>	<b>0.010</b>	BALL	0.305
BANK OF AMERICA	0.144	<b>BANK OF NEW YORK (21)</b>	<b>0.011</b>
BANK ONE	0.145	BARD C R	0.058
BAUSCH & LOMB	0.258	<b>BAXTER INTL. (22)</b>	<b>0.017</b>
BB & T	0.390	<b>BEAR STEARNS (23)</b>	<b>0.001</b>
BECTON DICKINSON & .CO.	0.097	<b>BED BATH &amp; .BEYOND (24)</b>	<b>0.000</b>
<b>BELLSOUTH (25)</b>	<b>0.026</b>	<b>BEMIS (26)</b>	<b>0.011</b>
BEST BUY CO.	0.970	BIG LOTS	0.403
BIOGEN IDEC	0.115	<b>BIOMET (27)</b>	<b>0.007</b>
<b>BJ SVS. (28)</b>	<b>0.008</b>	BLACK & .DECKER	0.325
H & R BLOCK	0.959	BMC SOFTWARE	0.266
BOEING	0.108	BOISE CASCADE	0.583
BOSTON SCIENTIFIC	0.343	<b>BRISTOL MYERS SQUIBB (29)</b>	<b>0.000</b>
BROWN - FORMAN 'B'	0.535	<b>BRUNSWICK (30)</b>	<b>0.002</b>
<b>BURL.NTHN.SANTA FE C (31)</b>	<b>0.002</b>	<b>BURLINGTON RES. (32)</b>	<b>0.002</b>
CAMPBELL SOUP	0.133	CARDINAL HEALTH	0.189
CARNIVAL	0.319	CATERPILLAR	0.119
CENDANT	0.569	<b>CENTERPOINT EN. (33)</b>	<b>0.000</b>
CENTEX	0.052	CENTURYTEL	0.152
<b>CHARLES SCHWAB (34)</b>	<b>0.019</b>	<b>CHARTER ONE FINL. (35)</b>	<b>0.000</b>
CHEVRONTEXACO	0.054	CHIRON CORP	0.861
<b>CHUBB (36)</b>	<b>0.011</b>	CIGNA	0.728
<b>CINCINNATI FIN. (37)</b>	<b>0.000</b>	CINTAS	0.070
CIRCUIT CITY STORES	0.334	CISCO SYSTEMS	0.143
<b>CITIGROUP (38)</b>	<b>0.019</b>	<b>CITIZENS COMMS. (39)</b>	<b>0.000</b>
CLEAR CHL.COMMS.	0.889	<b>CLOROX (40)</b>	<b>0.000</b>
CMS ENERGY	0.096	COCA COLA	0.224
COCA COLA ENTS.	0.239	<b>COLGATE - PALM. (41)</b>	<b>0.003</b>
<b>COMCAST 'A' (42)</b>	<b>0.039</b>	COMERICA	0.054
COMPUTER ASSOCS.INTL.	0.158	COMPUTER SCIS.	0.902

Table 6: Probability Values for S&amp;P 500 Series (COMPUWARE - ITT INDUSTRIES)

Company Name	P. Value	Company Name	P. Value
COMPUWARE	0.596	COMVERSE TECH.	0.258
CONAGRA	0.327	<b>CONCORD EFS (43)</b>	<b>0.042</b>
<b>CONOCOPHILLIPS (44)</b>	<b>0.021</b>	CONS.EDISON	0.495
CONSTELLATION EN.	0.565	<b>COOPER INDS. (45)</b>	<b>0.001</b>
COOPER TIRE RUB.	0.137	<b>ADOLPH COORS 'B' (46)</b>	<b>0.024</b>
CORNING	0.152	COUNTRYWIDE FINL.	0.554
CRANE	0.347	<b>CSX (47)</b>	<b>0.006</b>
CUMMINS	0.766	CVS	0.529
<b>DANA (48)</b>	<b>0.025</b>	DANAHER	0.722
DEERE & CO.	0.561	DELL	0.793
<b>DELTA AIR LINES (49)</b>	<b>0.000</b>	DELUXE	0.276
DILLARDS 'A'	0.414	DOLLAR GENERAL	0.415
DOMINION RES.	0.103	DONNELLEY R R	0.876
DOVER	0.687	DOW CHEMICALS	0.329
DOW JONES & .CO	0.059	<b>DTE ENERGY (50)</b>	<b>0.014</b>
<b>DU PONT E I DE NEMOURS (51)</b>	<b>0.000</b>	DUKE ENERGY	0.170
<b>DYNEGY 'A' (52)</b>	<b>0.000</b>	EASTMAN KODAK	0.667
EATON	0.366	<b>ECOLAB (53)</b>	<b>0.000</b>
<b>EDISON INTL. (54)</b>	<b>0.000</b>	<b>EL PASO (55)</b>	<b>0.000</b>
<b>ELECTRONIC ARTS (56)</b>	<b>0.011</b>	<b>ELECTRONIC DATA SYSTEMS(57)</b>	<b>0.000</b>
<b>EMC (58)</b>	<b>0.002</b>	<b>EMERSON ELECTRIC (59)</b>	<b>0.002</b>
<b>ENGELHARD (60)</b>	<b>0.001</b>	ENTERGY	0.882
EOG RES.	0.278	EQUIFAX	0.455
EXELON	0.865	<b>EXPRESS SCRIPTS 'A' (61)</b>	<b>0.023</b>
<b>EXXON MOBIL (62)</b>	<b>0.000</b>	FAMILY \$.STRS.	0.144
<b>FANNIE MAE (63)</b>	<b>0.000</b>	<b>FREDDIE MAC (64)</b>	<b>0.006</b>
FEDERATED DEPT.STRS.	0.428	FEDEX	0.333
FIFTH THIRD BANCORP	0.159	<b>FIRST DATA (65)</b>	<b>0.008</b>
<b>FIRST TEN.NAT. (66)</b>	<b>0.043</b>	FIRSTENERGY	0.487
<b>FISERV (67)</b>	<b>0.030</b>	<b>FLEETBOSTON FINL. (68)</b>	<b>0.000</b>
FORD MOTOR	0.393	FOREST LABS.	0.154
FORTUNE BRANDS	0.700	FPL GROUP	0.835
FRANK.RES.	0.209	GANNETT	0.079
GAP	0.368	GEN.DYNAMICS	0.346
<b>GENERAL ELECTRIC (69)</b>	<b>0.000</b>	GEN.MILLS	0.547
GENERAL MOTORS	0.840	GENUINE PARTS	0.065
GENZYME	0.213	GEORGIA PACIFIC	0.619
GILLETTE	0.341	<b>GOLDEN WEST FINL. (70)</b>	<b>0.039</b>
GOODRICH	0.063	GOODYEAR TIRE	0.384
GRAINGER W W	0.229	GT.LAKES CHM.	0.313
<b>HALLIBURTON (71)</b>	<b>0.000</b>	<b>HARLEY - DAVIDSON (72)</b>	<b>0.035</b>
HARRAHS ENTM.	0.675	<b>HASBRO (73)</b>	<b>0.001</b>
<b>HCA (74)</b>	<b>0.003</b>	<b>HEALTH MAN.AS.A (75)</b>	<b>0.034</b>
<b>HEINZ HJ (76)</b>	<b>0.019</b>	HERCULES	0.052
<b>HERSHEY FOODS (77)</b>	<b>0.004</b>	HEWLETT - PACKARD	0.268
HILTON HOTELS	0.940	<b>HOME DEPOT (78)</b>	<b>0.007</b>
HONEYWELL INTL.	0.920	<b>HUMANA (79)</b>	<b>0.007</b>
HUNTINGTON BCSH.	0.065	<b>ILLINOIS TOOL WKS. (80)</b>	<b>0.009</b>
INGERSOLL - RAND	0.641	INTEL	0.829
INTL.BUS.MACH.	0.174	INTL.FLAV.& FRAG.	0.681
INTL.GAME TECH.	0.218	INTL.PAPER	0.401
INTERPUBLIC GP.	0.099	ITT INDUSTRIES	0.182

Table 7: Probability Values for S&amp;P 500 Series (JP MORGAN CHASE - PULTE HOMES)

Company Name	P. Value	Company Name	P. Value
JP MORGAN CHASE & .CO.	0.471	JEFFERSON PILOT	0.365
<b>JOHNSON &amp; JOHNSON (81)</b>	<b>0.000</b>	JOHNSON CONTROLS	0.056
JONES APPAREL GROUP	0.867	<b>KB HOME (82)</b>	<b>0.017</b>
KELLOGG	0.318	KERR - MCGEE	0.060
KEYCORP	0.243	KEYSPAN	0.779
<b>KIMBERLY - CLARK (83)</b>	<b>0.000</b>	<b>KINDER MORGAN KANS (84)</b>	<b>0.000</b>
<b>KLA TENCOR (85)</b>	<b>0.001</b>	KNIGHT - RIDDER	0.374
<b>KOHL'S (86)</b>	<b>0.021</b>	<b>KROGER (87)</b>	<b>0.027</b>
LEGGETT& PLATT	0.134	LILLY ELI	0.584
LIMITED BRANDS	0.489	LINCOLN NAT.	0.686
<b>LINEAR TECH. (88)</b>	<b>0.025</b>	LIZ CLAIBORNE	0.689
<b>LOEWS (89)</b>	<b>0.042</b>	LNA.PACIFIC	0.500
LOWE'S COMPANIES	0.138	LSI LOGIC	0.325
MANOR CARE	0.225	<b>MARATHON OIL (90)</b>	<b>0.029</b>
<b>MARSH &amp; MCLENNAN (91)</b>	<b>0.000</b>	MARSHALL & ILSLEY	0.785
<b>MASCO (92)</b>	<b>0.043</b>	<b>MATTEL (93)</b>	<b>0.000</b>
<b>MAXIM INTEGRATED PRDS. (94)</b>	<b>0.000</b>	MAY DEPT.STORES	0.413
MAYTAG	0.188	<b>MBIA (95)</b>	<b>0.000</b>
<b>MBNA (96)</b>	<b>0.000</b>	<b>MCCORMICK &amp; .CO NV. (97)</b>	<b>0.000</b>
MCDONALDS	0.563	<b>MCGRAW - HILL CO. (98)</b>	<b>0.000</b>
MEADWESTVACO	0.171	MEDIMMUNE	0.541
<b>MEDTRONIC (99)</b>	<b>0.001</b>	MELLON FINL.	0.226
MERCK & .CO.	0.321	<b>MEREDITH (100)</b>	<b>0.044</b>
MERRILL LYNCH & .CO.	0.227	MGIC INVT	0.279
MICRON TECH.	0.813	MICROSOFT	0.201
<b>MILLIPORE (101)</b>	<b>0.032</b>	<b>MOLEX (102)</b>	<b>0.003</b>
<b>MOTOROLA (103)</b>	<b>0.006</b>	<b>NABORS INDS. (104)</b>	<b>0.011</b>
NAT.CITY	0.203	NATIONAL SEMICON.	0.368
NAVISTAR INTL.	0.853	NEW YORK TIMES 'A'	0.678
<b>NEWELL RUBBERMAID (105)</b>	<b>0.011</b>	NEWMONT MINING	0.602
NEXTEL COMMS.A	0.188	<b>NICOR (106)</b>	<b>0.000</b>
<b>NIKE 'B' (107)</b>	<b>0.010</b>	<b>NISOURCE (108)</b>	<b>0.040</b>
<b>NOBLE (109)</b>	<b>0.000</b>	NORDSTROM	0.612
NORFOLK SOUTHERN	0.203	<b>NORTH FORK BANCORP. (110)</b>	<b>0.018</b>
<b>NTHN.TRUST (111)</b>	<b>0.001</b>	NORTHROP GRUMMAN	0.525
NOVELL	0.836	<b>NOVELLUS SYSTEMS (112)</b>	<b>0.001</b>
NUCOR	0.230	OCCIDENTAL PTL.	0.721
OFFICE DEPOT	0.091	<b>OMNICOM GP. (113)</b>	<b>0.036</b>
ORACLE	0.177	<b>PACCAR (114)</b>	<b>0.027</b>
PALL	0.653	<b>PARAMETRIC TECH. (115)</b>	<b>0.009</b>
PARKER - HANNIFIN	0.091	<b>PAYCHEX (116)</b>	<b>0.014</b>
<b>PENNEY JC (117)</b>	<b>0.007</b>	PEOPLES ENERGY	0.856
<b>PEOPLESOFT (118)</b>	<b>0.021</b>	<b>PEPSICO (119)</b>	<b>0.032</b>
<b>PERKINELMER (120)</b>	<b>0.003</b>	<b>PFIZER (121)</b>	<b>0.046</b>
<b>PG &amp; .E (122)</b>	<b>0.038</b>	<b>PHELPS DODGE (123)</b>	<b>0.023</b>
PINNACLE WEST CAP.	0.304	<b>PITNEY - BOWES (124)</b>	<b>0.032</b>
<b>PLUM CREEK TIMBER (125)</b>	<b>0.001</b>	PMC - SIERRA	0.150
PNC FINL.SVS.GP.	0.063	PPG INDUSTRIES	0.126
PPL	0.245	<b>PRAXAIR (126)</b>	<b>0.044</b>
<b>PROCTER &amp; GAMBLE (127)</b>	<b>0.003</b>	PROGRESS EN.	0.784
PROGRESSIVE OHIO	0.086	<b>PROVIDIAN FINL. (128)</b>	<b>0.001</b>
PUB.SER.ENTER.GP.	0.245	PULTE HOMES	0.238

Table 8: Probability Values for S&amp;P 500 Series (QUALCOMM - 3M)

Company Name	P. Value	Company Name	P. Value
QUALCOMM	0.133	RADIOSHACK	0.092
RAYTHEON 'B'	0.491	REEBOK INTL.	0.424
<b>REGIONS FINL. (129)</b>	<b>0.023</b>	ROBERT HALF INTL.	0.443
<b>ROCKWELL AUTOMATION (130)</b>	<b>0.007</b>	ROHM & HAAS	0.183
ROWAN COS.	0.174	<b>RYDER SYSTEM (131)</b>	<b>0.033</b>
SAFECO	0.871	SAFEWAY	0.642
SARA LEE	0.802	<b>SBC COMMUNICATIONS (132)</b>	<b>0.013</b>
<b>SCHERING - PLOUGH (133)</b>	<b>0.004</b>	<b>SCHLUMBERGER (134)</b>	<b>0.000</b>
SCIENTIFIC ATLANTA	0.349	SEALED AIR	0.154
SEARS ROEBUCK & .CO.	0.838	SEMPRA EN.	0.076
SHERWIN - WILLIAMS	0.244	SIGMA ALDRICH	0.490
SLM	0.314	SNAP - ON	0.825
SOLETRON	0.665	SOUTHERN	0.219
SOUTHTRUST	0.253	SOUTHWEST AIRLINES	0.297
<b>SPRINT (135)</b>	<b>0.000</b>	<b>ST.JUDE MED. (136)</b>	<b>0.000</b>
ST.PAUL	0.343	STANLEY WORKS	0.270
<b>STAPLES (137)</b>	<b>0.014</b>	STARBUCKS	0.056
STARWOOD HTLS.& .RESORTS	0.059	STATE STREET	0.333
STRYKER	0.092	SUN MICROSYSTEMS	0.533
<b>SUNGARD DATA SYSTEMS (138)</b>	<b>0.019</b>	SUNOCO	0.712
SUNTRUST BANKS	0.385	SUPERVALU	0.488
<b>SYMANTEC (139)</b>	<b>0.023</b>	SYMBOL TECHS.	0.980
SYNOVUS FINL. (140)	0.004	<b>SYSCO (141)</b>	<b>0.000</b>
<b>T ROWE PRICE GP. (142)</b>	<b>0.001</b>	<b>TARGET (143)</b>	<b>0.004</b>
<b>TECO ENERGY (144)</b>	<b>0.000</b>	<b>TEKTRONIX (145)</b>	<b>0.001</b>
TELLABS	0.177	TEMPLE INLAND	0.307
<b>TENET HLTHCR. (146)</b>	<b>0.000</b>	TERADYNE	0.059
<b>TEXAS INSTS. (147)</b>	<b>0.003</b>	<b>TEXTRON (148)</b>	<b>0.001</b>
THERMO ELECTRON	0.145	THOMAS & .BETTS	0.063
TIFFANY & CO	0.936	TIME WARNER	0.855
TJX COS.	0.545	TORCHMARK	0.337
TOYS R US HOLDINGS CO.	0.676	TRIBUNE	0.942
<b>TXU (149)</b>	<b>0.000</b>	<b>TYCO INTL. (150)</b>	<b>0.000</b>
US BANCORP	0.220	<b>UNION PACIFIC (151)</b>	<b>0.013</b>
<b>UNION PLANTERS (152)</b>	<b>0.017</b>	UNISYS	0.988
<b>UNITEDHEALTH GP. (153)</b>	<b>0.037</b>	US.STEEL	0.485
<b>UNITED TECHNOLOGIES (154)</b>	<b>0.006</b>	<b>UNOCAL (155)</b>	<b>0.007</b>
UNUMPROVIDENT	0.397	UST	0.375
V F	0.534	<b>VERIZON COMMS. (156)</b>	<b>0.004</b>
<b>VIACOM 'B' (157)</b>	<b>0.021</b>	<b>VULCAN MATERIALS (158)</b>	<b>0.003</b>
WACHOVIA	0.079	<b>WALGREEN (159)</b>	<b>0.001</b>
<b>WAL MART STORES (160)</b>	<b>0.001</b>	WALT DISNEY	0.491
WASHINGTON MUTUAL	0.212	WASTE MAN.	0.076
<b>WELLS FARGO &amp; .CO (161)</b>	<b>0.035</b>	WENDY'S INTL.	0.120
WEYERHAEUSER	0.355	WHIRLPOOL	0.530
<b>WILLIAMS COS. (162)</b>	<b>0.000</b>	<b>WINN - DIXIE STRS. (163)</b>	<b>0.000</b>
WORTHINGTON INDS.	0.089	WRIGLEY WILLIAM JR.	0.190
<b>WYETH (164)</b>	<b>0.002</b>	XCEL ENERGY	0.275
XEROX	0.809	XILINX	0.120
ZIONS BANCORP.	0.272	<b>3M (165)</b>	<b>0.001</b>

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**Department of Economics  
Queen Mary, University of London  
Mile End Road  
London E1 4NS  
Tel: +44 (0)20 7882 5096  
Fax: +44 (0)20 8983 3580  
Web: [www.econ.qmul.ac.uk/papers/wp.htm](http://www.econ.qmul.ac.uk/papers/wp.htm)**