School of Economics and Finance

Competition of E-Commerce Intermediaries

Alexander Matros and Andriy Zapechelnyuk

Working Paper No. 675 November 2010 ISSN 1473-0278

Queen Mary
University of London

Competition of E-Commerce Intermediaries*

Alexander Matros†
Moore School of Business, University of South Carolina

Andriy Zapechelnyuk‡
Queen Mary, University of London

November 13, 2010

Abstract

In e-commerce, where information collection is essentially costless and geographic location of traders matters very little, fierce competition between providers of similar services is expected. We consider a model where two e-commerce intermediaries (internet shops) compete for sellers. We show that two non-identical shops may coexist in equilibrium if the population of sellers is sufficiently differentiated in their time preferences. In such an equilibrium less patient sellers choose the more popular (with a higher rate of arrival of new buyers) and more expensive shop, while more patient sellers prefer the less popular and cheaper one.

Keywords: E-commerce, intermediary, competition, listing fee, closing fee

JEL Classification: C73, D43, D82

*We thank Philip A. Haile, Benny Moldovanu, and seminar participants at the University of Bonn for their comments.

†Department of Economics, Moore School of Business, University of South Carolina, SC, USA. *E-mail:* `alexander.matros` $\alpha\tau$ `gmail.com`

‡School of Economics and Finance, Queen Mary, University of London, Mile End Road, London E1 4NS, UK. E-mail: `a.zapechelnyuk` $\alpha\tau$ `qmul.ac.uk`

1 Introduction

In the modern world buyers and seller increasingly rely on the internet where information about products and prices is easily collected and geographic boundaries stop playing a significant role. The possibility to trade through websites led to a boom on the secondary market where people trade second-hand electronic devices, household objects, etc. If in old times a person who wished to buy some used item would have to visit a local flee market, search through garage sales, or read local newspaper ads, nowadays this can be done by a few mouse-clicks. Current internet marketplaces are big bazaars that pool buyers and sellers without respect for geographical locations, where each buyer is likely to find exactly the product that she is looking for. Examples of such marketplaces are abound, including Cellbazaar, Amazon, eBay, Yahoo among many others.

There are two main features that differentiate virtual marketplaces from traditional ones. First, in a traditional market a seller is usually charged only a flat fee for using a trading place during some period of time; more elaborate fees that depend on sales or revenue are difficult to monitor and enforce. In contrast, in internet transactions a lot more information is recorded, so service fees can depend on more variables, in particular, on the amount of revenue the seller receives. So, the first question we address is:

What is the optimal structure of service fees for an e-commerce intermediary?

The second feature is that it is costless to list and display a product at internet shops. Moreover, a geographical location of a seller does not matter for listing a product (it may matter only afterwards, when the product is sold, in terms of shipping costs). This background stimulates fierce competition among internet shops, until the strongest survives. This is the case for eBay which took over more than fifteen internet auction companies that operated on local as well as international markets in the last twelve years[1]

[1] The list of eBays acquisitions includes Up4Sale.com, Butterfield & Butterfield, Alando (Germany), Half.com, Internet Auction Co. (South Korea), iBazar (France), CARad.com, EachNet (China), Baazee.com (India), Marktplaats.nl (Netherlands), Shopping.com (US), Tradera (Sweden), StubHub (US), GittiGidiyor (Turkey), StumbleUpon (Canada), Afterbuy (Germany).

and driven out of market others.[2] However, there are many competing intermediaries in some specific internet markets. For example, Amazon competes with Barnes & Noble, as well as with other smaller shops on the internet book market. Thus the second question that we address is:

> *What is the equilibrium market structure for competing internet intermediaries? Will one e-commerce company eventually monopolize the market, or can competitors coexist in equilibrium?*

We model two e-commerce intermediaries (internet shops) that compete for sellers. Each shop charges sellers two service fees: a *listing fee*, a fixed amount paid by a seller in every period for having his object listed, and a *closing fee*, a percentage of the price after the object has been sold. The fees are announced at the beginning of the game and stay fixed thereafter. Sellers are drawn at random from a heterogeneous population. A seller observes the service fees and then, repeatedly in discrete periods, decides at which shop and at what price to list his object. Each period buyers arrive to internet shops according to a Bernoulli process. We assume that internet shops differ in the probability of buyers' arrival.[3] A buyer purchases the listed product if her willingness to pay exceeds the listed price; otherwise she leaves and never returns. The game ends after the object has been sold (or consumed by the seller himself).

Under the assumption that internet shops are at least as patient as sellers (in their preferences over time), our analysis provides a new insight regarding optimal selection of service fees. We show that a positive listing fee is never optimal, and, consequently, *in any equilibrium the listing fee must be equal to zero.* The intuition behind this result is as follows. When a shop is more patient than a seller, due to a distortion between their interests the price chosen by the seller is too low from the perspective of the shop; the

[2]eBay's main competitors, Yahoo and Amazon, discontinued their Internet auction service, Yahoo on June 16, 2007 and Amazon on September 8, 2008.

[3]Empirical observations support our assumption about different popularity of internet shops – see the literature on the market structure of e-commerce (e.g., Brown and Morgan, 2009; Goldmanis et al., 2010). The search literature also provides several models where different shops will have different popularity (e.g., Stahl, 1989, 1996).

distortion is minimized when the listing fee is zero. Thus, we show that a shop's policy is effectively one-dimensional: it is the choice of its closing fee only.

Second, we characterize Bayesian Nash equilibria in our model and show that there exists at most one equilibrium which, depending on the model parameters, may be one of the following types: *monopoly*, *contestable monopoly* or *market segmentation*. An internet shop becomes a monopoly if it is so popular (relative to the competitor) that sellers of all types prefer it even if the monopoly fees are charged. A contestable monopoly equilibrium arises if shops are not too different in their popularity and the pool of sellers is not too differentiated. It is a result of a standard Bertrand competition, where a more popular internet shop (with a higher rate of buyers' arrival) sets fees so low that sellers of every type will be attracted to that shop, and the competitor is forced to leave the market.

Monopoly or contestable monopoly are, probably, most expected market structures. However, if sellers are sufficiently differentiated, a *market segmentation* equilibrium obtains, where both internet shops receive positive profit. This situation takes place if the more popular shop can obtain higher expected payoffs by attracting only sellers of a specific type (rather than all types, as it is in the contestable monopoly). This allows its competitor to set low positive fees, attract sellers of a different type, and obtain a positive expected payoff as well. This equilibrium is a result of "seller's differentiation" where sellers are discriminated on the basis of their time preferences. Less patient sellers are naturally attracted to the more popular shop that provides a larger number of buyers in a short while. In contrast, more patient sellers are not so constrained by time, so they can afford to wait longer for a successful sale by listing the product at the less popular but cheaper internet shop. This equilibrium resembles classical results for an oligopoly with differentiated products (e.g., Shaked and Sutton, 1982; Singh and Vives, 1984), except in our model products are identical, but sellers are differentiated in their time preferences.

Finally, in Section 6 we show that the results can be extended to more general trade mechanisms, including, most notably, internet auctions, where objects are sold via a Vickrey auction. Ellison, Fudenberg, and Möbius (2004) in a related article on competing auctions (see also Moldovanu, Sela, and Shi, 2008) find that whenever two auction intermediaries co-exist in equilibrium, the "law of one price" (i.e., the same expected closing price and the same buyer/seller ratio across competing auctions) should hold. This result relies on the assumption that bidders can freely choose between auction houses. Contrarily to Ellison, Fudenberg, and Möbius (2004), Brown and Morgan (2009) demonstrate in field experiments that eBay prices were consistently higher than those on Yahoo, and eBay attracted more buyers per seller. Our *market segmentation* equilibrium suggests an explanation for Brown and Morgan's (2009) findings.

In a related paper, Baye and Morgan (2001) consider a model with one profit-maximizing intermediary. They are interested in the price dispersion phenomenon on the internet markets and in overall welfare effects of introducing internet shopping. Baye and Morgan's (2001) framework has a number of differences from ours. Most notably, there is a monopoly intermediary in their model, while we analyze competitive internet shops; the interaction in their model is one-shot, while we have a multi-period trade; their intermediary can charge only listing fee from the sellers, while we allow for a two-dimensional fee structure.

The paper is organized as follows. The model is described in Section 2. In Sections 3 and 4 we derive the optimal behavior of a representative seller and the competing internet shops. Section 5 characterizes all equilibria. In Section 6 we extend our basic model to allow for more general trade mechanisms. All proofs are deferred to the Appendix.

2 The Model

Let $\mathcal{N}$ be a large (infinite) population of buyers and $\mathcal{M}$ be a large (infinite) population of sellers. Every buyer $i \in \mathcal{N}$ is characterized by her private *use value* $v_i \in [\underline{v}, \overline{v}]$ of the object. All private use values are independent and identically distributed according to distribution function F. Every seller $s \in \mathcal{M}$ is characterized by two independent private parameters: his use value $v_s \in [\underline{v}, \overline{v}]$ and discount factor δ_s. We assume sellers' use values are independent and identically distributed according to distribution function G and that δ_s has either low (δ_L) or high (δ_H) value with probability α and $(1-\alpha)$ respectively, where

$0 < \delta_L \leq \delta_H < 1$ and $\alpha \in [0, 1]$. We also assume that functions F and G are differentiable and have positive density on $(\underline{v}, \overline{v})$, and, in addition, satisfy the monotonic hazard rate conditions: $\frac{f(z)}{1-F(z)}$ and $\frac{g(z)}{G(z)}$ are strictly increasing on $(\underline{v}, \overline{v})$, where f and g denote the corresponding density functions.[4]

The timing of the game is as follows. In period 0, two intermediaries (internet shops) $j = 1, 2$ simultaneously announce sellers' fees for all subsequent transactions: listing fees, $c_j \geq 0$, and closing fees, a fraction $\mu_j \in [0, 1]$ of the price. The fees are commitments that cannot be altered in later periods.[5] In discrete periods $t = 1, 2, \ldots$ buyers arrive to shops and observe what is on sale. Each shop j is characterized by the rate of arrival of buyers, the probability η_j that a new buyer arrives in every period. The shops may have different arrival rates buyers. If $\eta_1 > \eta_2$, that is, shop 1 expects more buyers to show up in any fixed time frame, then we say that shop 1 is *more popular*.

After the service fees have been announced, a representative seller is drawn from population $\mathcal{M}$. Then, in each period $t = 1, 2, \ldots$ the seller chooses a shop j to list his product and a price to be listed (he may switch between shops or update the price freely after every period). Alternatively, he decides to consume the object (and thus receive the payoff equal to the object's use value v_s). If in period t the object is listed in shop j, with probability η_j a buyer (randomly drawn from population $\mathcal{N}$) arrives, and purchases the object if and only if her use value is greater than the price; otherwise she leaves and never returns. Regardless of the outcome, the seller pays to shop j the listing fee, c_j, and, in addition, if the object is sold, the closing fee, fraction μ_j of the price. If the object is purchased or consumed, the game ends; otherwise it proceeds to the next period.

Distribution functions F and G and parameters α, δ_L, δ_H, η_1 and η_2 are common knowledge. We also assume that all players are risk neutral.

[4]This is a standard assumption in the literature (e.g., Myerson and Satterthwaite, 1983; Krishna, 2002).

[5]It is important that the listing fees are nonnegative, that is, internet shops only collect fees from sellers and do not pay to sellers. This constraint is naturally satisfied in real life. Trade intermediaries sell products *on behalf* of the owners, charge fees for the service, and do not purchase any products.

3 Seller's Decision Problem

In this section we describe and solve the seller's decision problem. Consider a seller with use value $v_s \in [\underline{v}, \overline{v}]$ and discount factor $\delta_s \in \{\delta_L, \delta_H\}$. Given the stationary environment, we will consider seller's time-invariant (Markov) strategies.

The seller observes the fees of both shops, (c_1, μ_1) and (c_2, μ_2), and makes the following decisions. First, given the fees, he calculates his expected revenue from using shop 1 and shop 2. Then, he makes an allocation decision (λ_1, λ_2), where λ_j is the probability that the object is listed at shop $j = 1, 2$, and $1 - \lambda_1 - \lambda_2$ is the probability that the object is consumed, $\lambda_1, \lambda_2 \in [0, 1]$, $\lambda_1 + \lambda_2 \leq 1$.

Let us calculate the seller's expected revenue from using shop j. Suppose that the seller has chosen shop j for listing the object. For every seller's price p_j, denote by $Q_j(p_j)$ the *probability that the object is sold* in shop j in the following period, so we have

$$Q_j(p_j) = (1 - F(p_j))\eta_j,$$

where η_j is the probability that a buyer arrives to the shop and $1-F(p_j)$ is the probability that her use value is above p_j. Denote by $u_j(p_j, u^*)$ the seller's expected revenue from using shop j, where u^* is the highest continuation payoff that the seller expects to obtain in the next period if the object is not sold.[6] Thus,

$$u_j(p_j, u^*) = -c_j + Q_j(p_j) \cdot (1 - \mu_j)p_j + (1 - Q_j(p_j)) \cdot \delta_s u^*. \tag{1}$$

Then, an optimal price $p_j(u^*)$ is a solution of the optimization problem

$$p_j(u^*) \in \arg\max_{p_j \in [\underline{v}, \overline{v}]} u_j(p_j, u^*). \tag{2}$$

Lemma 1 *For every pair of fees (c_j, μ_j) and every $u^* \in [\underline{v}, \bar{v}]$, there exists a unique solution $p_j(u^*) \in [\underline{v}, \bar{v}]$ of decision problem* (2)*. It is the solution of the following equation*

$$p_j - \frac{1 - F(p_j)}{f(p_j)} = \frac{\delta_s u^*}{1 - \mu_j}, \ \textit{if } \delta_s u^* < (1 - \mu_j) \left(\bar{v} - \frac{1 - F(\bar{v})}{f(\bar{v})} \right), \tag{3}$$

[6] We assume that the seller does not derive any utility from the object before it is sold or consumed.

and

$$p_j = \bar{v}, \text{ if } \delta_s u^* \geq (1-\mu_j)\left(\bar{v} - \frac{1-F(\bar{v})}{f(\bar{v})}\right).$$

Let us now consider the seller's allocation decision (λ_1, λ_2). The seller makes an allocation decision (to consume the object or to list it at shop 1 or 2) which maximizes his expected payoff. Note that since in every period the seller faces ex-ante the same environment, if he chooses shop j once, then he will choose it in all periods until the object is sold.

Let u_j^* be the expected seller's payoff if he tries to sell the object at shop j in all periods. From (1) it follows that u_j^* is a solution of the following equation,

$$\begin{aligned} u_j^* &= u_j(p_j(u_j^*), u_j^*) \\ &\equiv \left(-c_j + (1-\mu_j)Q_j(p)p + (1-Q_j(p))\delta_s u_j^*\right)\big|_{p=p_j(u_j^*)}. \end{aligned} \tag{4}$$

The next lemma states that equation (4) has a unique solution.

Lemma 2 *The mapping $u_j(p_j(\cdot), \cdot)$ has a unique fixed point.*

The seller chooses $(\lambda_1^*, \lambda_2^*)$ to maximize his expected revenue

$$(\lambda_1^*, \lambda_2^*) \in \arg\max_{\lambda_1, \lambda_2} \lambda_1 u_1^* + \lambda_2 u_2^* + (1-\lambda_1-\lambda_2)v_s. \tag{5}$$

We say that a seller *prefers* shop i to shop j if $u_i^* \geq u_j^*$. Thus, the seller lists the object at a preferred shop and receives $\max\{u_1^*, u_2^*\}$ whenever this revenue exceeds his use value v_s, otherwise, he consumes the object and receives v_s.

Observe that the expected payoff from shop j, u_j^*, and the optimal price $p_j(u_j^*)$ do not depend on the seller's use value, v_s, but they depend on the seller's discount factor δ_s. Indeed, a more patient seller chooses a higher price and receives a higher expected revenue after listing the object. Therefore, sellers with the same discount factor have the same preference over shops, and only sellers with different discount factors may prefer different shops. With a slight abuse of terminology, we will refer to a more (less) patient seller with discount factor, δ_H (δ_L), as an *H-type* (respectively, *L-type*) seller.

4 Optimal Fees

A seller's behavior as a function of the shops' fees was described in the previous section. In this section, we consider how shops choose their fees.

4.1 Payoffs

Consider shop $j = 1, 2$. The probability that a new buyer arrives is $\eta_j > 0$ in every period. We assume that both shops have the same discount factor $\gamma \in (0, 1]$ and are more patient than the sellers, $\gamma \geq \delta_H (\geq \delta_L)$.[7] It is standard in the literature to assume that a firm (an internet shop) is more patient than an individual (a seller).

Fix shops' fees $a_1 = (c_1, \mu_1)$ and $a_2 = (c_2, \mu_2)$, the seller's use value v_s and his discount factor δ_θ, $\theta \in \{L, H\}$. Denote by $\lambda_j^\theta(a_1, a_2)$ the probability that a θ-type seller chooses shop j, by $u_j^\theta(a_j)$ the seller's expected payoff from listing the object with shop j, and by $p_j^\theta(a_j)$ the optimal selling price, $j = 1, 2$. The expected payoff of shop j conditional on that a θ-type seller has chosen for listing his object is

$$w_j^\theta(a_j) = c_j + Q_j(p_j^\theta(a_j)) \cdot \mu_j p_j^\theta(a_j) + (1 - Q_j(p_j^\theta(a_j))) \cdot \gamma w_j^\theta(a_j). \tag{6}$$

The next lemma shows that the above equation has a unique solution.

Lemma 3 *For every $a_j = (c_j, \mu_j) \in \mathbb{R}_+ \times [0, 1]$ and every $\theta \in \{L, H\}$, there exists a unique solution, $w_j^\theta(a_j)$, of equation* (6)*.*

The proof is similar to the proof of Lemma 2 and thus omitted.

The unconditional payoff $\bar{w}_j(a_1, a_2)$ of shop j is given by the product of the conditional payoff described above and the probability that a seller of each type chooses shop j for selling the object

$$\bar{w}_j(a_1, a_2) = \alpha \lambda_j^L(a_1, a_2) w_j^L(a_j) + (1 - \alpha) \lambda_j^H(a_1, a_2) w_j^H(a_j). \tag{7}$$

[7] The assumption that the shops share the same discount factor is not essential for the results and made for convenience.

Equation (7) shows that each shop faces the following trade-off: lower fees lead, on the one hand, to a higher probability of attracting a seller (or even to stealing a seller from the competitor), but, on the other hand, to a lower revenue from the transaction.

4.2 Listing Fees

This section addresses and partially answers one of the main questions of this paper: *What are the optimal fees?* We will show that if a shop makes a positive profit in an equilibrium, the listing fee in that equilibrium must be zero.

Theorem 1 *Let (c_1, μ_1) and (c_2, μ_2) be arbitrary fees of the shops. If shop i receives a positive expected payoff and $c_i > 0$, then (c_i, μ_i) is not a best reply of shop i to the competitor's fees (c_j, μ_j), $j \neq i$.*

The sketch of the proof is as follows. Suppose that a seller has chosen shop i. Since the seller is less patient than the shop, he will always choose his selling price lower than the price which maximizes the expected payoff of the shop.[8] Since $c_i > 0$, shop i can change the fees: increase closing fee μ_i and decrease listing fee c_i, such that the seller's expected revenue does not change, but his "virtual continuation value" $\frac{\delta_s v_s}{1-\mu_i}$ increases. By equation (3), a higher "virtual continuation value" leads to a higher selling price, which in turn increases the expected revenue of the shop.

It immediately follows from Theorem 1 that if a shop has a positive expected payoff in an equilibrium, then its listing fee must be equal to zero. However, in an equilibrium where shop i has zero payoff (and thus no deviation can lead to a positive payoff), every fee (c_i, μ_i) is a best reply, that is, all strategies of shop i lead to the same zero payoff. To simplify notations and characterization of equilibria, we will assume that if shop i has zero payoff in an equilibrium, then $c_i = 0$. This assumption and Theorem 1 give the following corollary.

[8]The assumption that the shops are at least as patient as sellers is important. The result in Theorem 1 need not hold if a sufficient fraction of the sellers' population is more patient than the shops.

Corollary 1 *In every equilibrium the listing fees of both shops are equal to zero.*

Corollary 1 allows us to fix listing fees at zero and describe the shops' strategies as the choice of *closing fees only.* In what follows, we will assume that the listing fees are zero. Thus, the notation for strategy $a_j = (c_j, \mu_j)$ of shop j will be replaced by μ_j and understood as $a_j = (0, \mu_j)$. Therefore, from (7), the unconditional payoff $\bar{w}_j(a_1, a_2) = \bar{w}_j(\mu_1, \mu_2)$ of shop j can be rewritten as

$$\bar{w}_j(\mu_1, \mu_2) = \alpha\lambda_j^L(\mu_1, \mu_2)w_j^L(\mu_j) + (1-\alpha)\lambda_j^H(\mu_1, \mu_2)w_j^H(\mu_j). \tag{8}$$

4.3 Closing Fees

We say that shop i is more *popular* than shop j if it attracts more buyers, i.e., $\eta_i > \eta_j$. Without loss of generality, we assume that $\eta_1 \geq \eta_2$.

Since the expected seller's revenue depends on the probability that a new buyer arrives, it is easy to see that selling an object in the more popular shop yields a higher selling price. Thus, if two shops charge equal fees, any seller (whether H-type or L-type) will prefer the more popular shop, and hence the less popular one attracts no sellers and receives zero payoff.

Suppose that the shops are equally popular, $\eta_1 = \eta_2$. Since in an equilibrium shops compete only in closing fees, they are engaged in the classic Bertrand competition: a shop with a lower closing fee is more attractive to a seller of *any* type. It immediately follows that closing fees must be equal to zero in an equilibrium.

Proposition 1
(i) If the shops are equally popular, $\eta_1 = \eta_2$, then there exists a unique equilibrium where all fees are equal to zero, $(c_1, \mu_1) = (0, 0) = (c_2, \mu_2)$.
(ii) If shop 1 is more popular, $\eta_1 > \eta_2$, then it sets a positive closing fee in equilibrium, $\mu_1 > 0$.

Part (i) of Proposition 1 characterizes completely the equilibria in the case of equally popular shops. In what follows, we will analyze a more complicated case, $\eta_1 > \eta_2$. In order to understand how the sellers of different types choose between the shops that are not equally popular, consider the following diagram (Fig. 1).

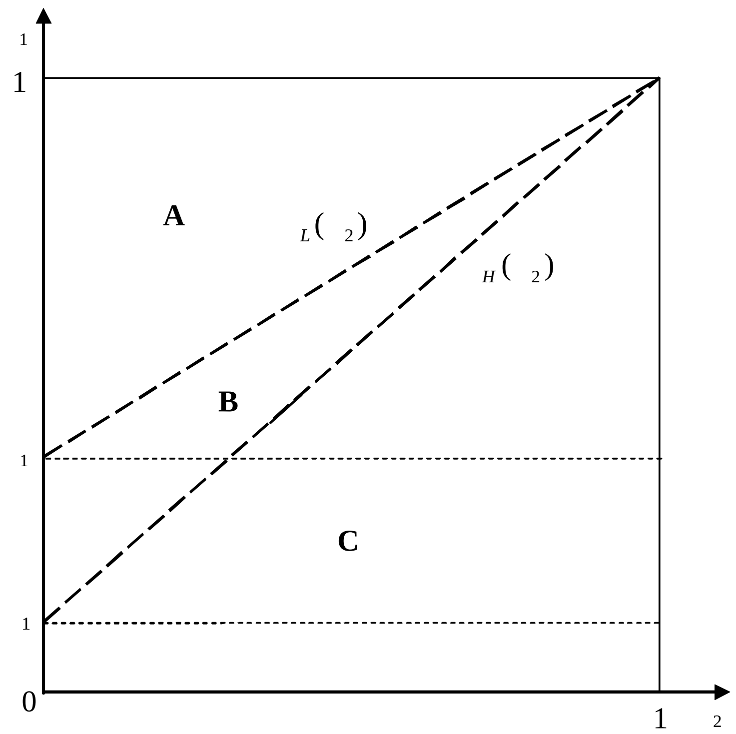

Figure 1: Indifference curves of L and H types of sellers

Figure 1 illustrates the indifference curves of L and H types of sellers for the case of $\eta_1 > \eta_2$. For every closing fee μ_2 of shop 2, let $\phi_\theta(\mu_2)$ denote the critical level of μ_1 such that with these fees a θ-type seller, $\theta = L, H$, is indifferent between the two shops. The graph $\{(\phi_\theta(\mu_2), \mu_2) : \mu_2 \in [0,1]\}$ represents the indifference curve of a θ-type seller. Note that at the point $(\mu_1, \mu_2) = (1, 1)$ the indifference curves for both seller types coincide, since in this case the shops claim the entire surplus, leaving sellers with zero expected revenue, regardless of their types. Two curves $\phi_L(\mu_2)$ and $\phi_H(\mu_2)$ divide the closing fee space, $\mu_1 \times \mu_2$, into three areas:

A is the area where μ_1 is too high relative to μ_2, so that all sellers prefer shop 2;

B is the area where sellers with different patience levels prefer different shops;

C is the area where μ_2 is too high relative to μ_1, so that all sellers prefer shop 1.

It is important to note that more popular shop 1 can always guarantee to attract L-type sellers by setting its closing fee $\mu_1 \leq \mu_1''$. Furthermore, it can always guarantee to attract *all* sellers by setting its closing fee $\mu_1 \leq \mu_1'$, no matter what closing fee μ_2 is chosen by shop 2. In contrast, less popular shop 2 cannot guarantee to attract any type of sellers.

It is clear from Figure 1 that if an H-type seller is indifferent between two shops, then L-type seller prefers more popular shop 1.

Proposition 2 *If $\eta_1 > \eta_2$, then $\phi_H(\mu_2) < \phi_L(\mu_2)$ for every $\mu_2 < 1$.*

Intuitively, for an L-type seller (the impatient one), the possibility to obtain a higher revenue *right now* is the dominant factor, and thus he receives a higher payoff from the more popular shop. To see this, imagine the extremely impatient seller, $\delta_L = 0$, who obtains utility only from the current-period sale. In this case, the probability of a new buyer arrival is of extreme importance for him. In contrast, a more patient seller can afford to wait and try to sell the object in more than one period, thus, eventually having a high probability of sale over time, even if the probability of a new buyer arrival is small in every period.

Thus, depending on model parameters, we might find that equilibrium fees are in area **C**, so that only the more popular shop is present on the market, or in area **B**, so that both shops are active and receive profit. It is clear that equilibrium fees cannot be in area **A**, since the more popular shop can always choose a lower fee and attract one or both types of sellers.

5 Equilibrium Analysis

In this section we will show that if shops are not equally popular, $\eta_1 > \eta_2$, then there are three types of equilibria: *monopoly*, *contestable monopoly*, and *market segmentation*.

5.1 Monopoly

Suppose that shop 1 is much more popular than shop 2, so that shop 1 is a monopoly on the market. For illustration, imagine the extreme case, $\eta_2 = 0$. In this extreme case a seller with use value v_s and discount factor δ_θ, $\theta = H, L$, will never choose shop 2 for any (μ_1, μ_2). Furthermore, the seller will choose shop 1 (as opposed to consuming the object) if and only if $u_1^\theta(\mu_1) > v_s$.[9] Hence, for a given closing fee μ_1, the probability that a seller of type θ lists the object is equal to the probability that $v_s < u_1^\theta(\mu_1)$, that is, for every $\theta = H, L$ and every μ_1

$$\lambda_1^\theta(\mu_1, \mu_2) = G(u_1^\theta(\mu_1)).$$

From (8), the expected payoff of the monopolist is

$$\begin{aligned}\bar{w}_1(\mu_1, \mu_2) &= \alpha\lambda_1^L(\mu_1, \mu_2)w_1^L(\mu_1) + (1-\alpha)\lambda_1^H(\mu_1, \mu_2)w_1^H(\mu_1) \\ &= \alpha w_1^L(\mu_1)G(u_1^L(\mu_1)) + (1-\alpha)w_1^H(\mu_1)G(u_1^H(\mu_1)).\end{aligned}$$

Therefore, shop 1 solves the following maximization problem

$$w^M = \max_{\mu_1}\left[\alpha w_1^L(\mu_1)G(u_1^L(\mu_1)) + (1-\alpha)w_1^H(\mu_1)G(u_1^H(\mu_1))\right]. \tag{9}$$

The following lemma helps to establish uniqueness of the solution.

Lemma 4 *For every $i = 1, 2$ and every $\theta = H, L$, the expression*

$$w_i^\theta(\mu_i) \cdot G(u_i^\theta(\mu_i))$$

is strictly concave in μ_i.

[9] A tie, $v_s = u_1^\theta(\mu_1)$, is a zero-probability event and thus can be ignored in the analysis.

A solution of (9) is unique, since the sum of concave functions is concave. We will refer to this solution, denoted by $\mu_1^M \in [0,1]$, as the *monopoly closing fee*. Further, the equilibrium where shop 1 sets the monopoly closing fee and attracts all sellers will be called the *monopoly equilibrium.*

Proposition 3 *Let $\eta_1 > \eta_2$. The monopoly equilibrium exists if and only if*

$$\mu_1^M \leq \phi_H(0). \tag{10}$$

5.2 Contestable Monopoly

Let $\eta_1 > \eta_2 > 0$. Consider an equilibrium where more popular shop 1 sets a positive closing fee less than the monopoly fee, attracts sellers of all types, and receives a positive expected payoff, while shop 2 attracts no sellers and receives zero payoff. This situation is a *contestable monopoly*: more popular shop 1 is a monopolist who is forced to set the closing fee low enough (lower than the monopoly closing fee) to keep the other shop from "entering the market" (setting a closing fee above zero) and obtaining a positive expected payoff. In a contestable monopoly equilibrium more popular shop 1 sets $\mu_1 = \phi_H(0) < \mu_1^M$ and attracts both types of sellers, i.e., $\lambda_2^\theta(\mu_1, \mu_2) = 0$ for each $\theta = H, L$.

Denote by μ_i^θ the monopoly fee for shop i who faces the population of sellers consisting of θ type only,

$$\mu_i^\theta = \arg\max_{\mu_i} w_i^L(\mu_i) \cdot G(u_i^\theta(\mu_i)), \quad \theta = H, L, \;\; i = 1, 2. \tag{11}$$

Proposition 4 *Let $\eta_1 > \eta_2$. A contestable monopoly equilibrium exists if and only if*

$$\mu_1^M > \phi_H(0) \tag{12}$$

and

$$\bar{w}_1(\phi_H(0), 0) \geq \bar{w}_1(\min\left\{\phi_L(0), \mu_1^L\right\}, 0). \tag{13}$$

Proposition 4 demonstrates that if shop 1 is more popular than shop 2 and cannot charge closing fee above $\phi_H(0)$ (because otherwise H-type and maybe even L-type sellers would switch to shop 2) a contestable monopoly equilibrium arises with the closing fees $(\mu_1, \mu_2) = (\phi_H(0), 0)$ and $\lambda_2^\theta(\phi_H(0), 0) = 0$ for each $\theta = H, L$. Condition (13) will be discussed in further details below.

5.3 Market Segmentation

(Contestable) monopoly is a situation that is natural to see in our model where two different shops compete in service fees for sellers. However, we show that an equilibrium outcome may be different.

Suppose that condition (13) does not hold. It means that shop 1 can obtain a higher payoff if it attracts *only* L-type sellers (achieved by charging a closing fee above $\phi_H(0)$) than if it attracts *both* types of sellers (achieved by charging closing fee equal to $\phi_H(0)$). This can happen, for instance, when the mass of H-type sellers in the population is small enough. Now, shop 2 can also raise its closing fee to collect a positive revenue from H-type sellers. Thus, the market is split into two segments where each shop attracts one type of sellers and receives positive expected profit. An equilibrium where L-type sellers prefer more popular shop 1 and H-type sellers prefer less popular shop 2 will be called a *market segmentation equilibrium.*[10]

Proposition 5 *In a market segmentation equilibrium the following holds:*

(i) *Each shop sets a monopoly fee on its market segment, i.e.,* $\mu_1 = \mu_1^L$ *and* $\mu_2 = \mu_2^H$*, and the sellers prefer the respective shop,*

$$\phi_H(\mu_2^H) < \mu_1^L < \phi_L(\mu_2^H)$$

and

$$\phi_L^{-1}(\mu_1^L) < \mu_2^H < \phi_H^{-1}(\mu_1^L).$$

[10] Note that the opposite situation, where the L-type sellers prefer less popular shop 2 and the H-type sellers prefer more popular shop 1, is impossible by Proposition 2.

(ii) *A payoff of shop j depends only on its own fees and is given by*

$$\bar{w}_1 = \alpha \max_{\mu_1} w_1^L(\mu_1) \cdot G(u_1^L(\mu_1))$$

and

$$\bar{w}_2 = (1-\alpha) \max_{\mu_2} w_2^H(\mu_2) \cdot G(u_2^H(\mu_2)).$$

Part (i) of the proposition shows that a market segmentation equilibrium can exist only inside area **B** (Figure 1) and part (ii) shows that each shop receives the monopoly payoff on the respective market segment. Note that since the monopoly fees μ_1^L and μ_2^H are unique, if a market segmentation equilibrium exists, it must be unique.

5.4 Characterization of Equilibria

The following theorem summarizes the above results in Section 5 and completes the characterization of equilibria.

Theorem 2 *There exists at most one equilibrium.*
If $\eta_1 = \eta_2$, then equilibrium closing fees are $(\mu_1, \mu_2) = (0, 0)$.
If $\eta_1 > \eta_2$, then the equilibrium is either monopoly, contestable monopoly, or market segmentation equilibrium.

Theorem 2 demonstrates that if an equilibrium exists, it must be one of the types we have discussed. Furthermore, it is unique[11] since each of the equilibrium types is uniquely defined. Note that since conditions for existence of the monopoly, contestable monopoly, and market segmentation equilibrium are mutually exclusive and do not cover the entire set of parameters, it follows that generally an equilibrium (in pure strategies) need not exist.

[11] Given our assumption that if a shop receives zero profit in equilibrium, it sets zero fees.

6 General Trade Mechanisms and Auctions

So far we considered the model with a simple trade mechanism where the seller posts a price and every buyer can only take it or leave it. In this section we will show that our results extend to a more general class of trade mechanisms that includes, in particular, Vickrey auction.

Let us call p a *reserve price*, interpreted as the seller's declaration of the amount that he is willing to accept in exchange of his product. A general trade mechanism of intermediary j is characterized by a pair (Q_j, R_j), $Q_j : [\underline{v}, \overline{v}] \to [0, 1]$ and $R_j : [\underline{v}, \overline{v}] \to \mathbb{R}$, where for every reserve price $p \in [\underline{v}, \overline{v}]$, $Q_j(p)$ is the probability that the object is sold in the current period and $R_j(p)$ is the expected revenue from sale. We impose the following constraints on Q_j and R_j:

(a) $Q_j(p)$ is weakly decreasing on $[\underline{v}, \overline{v}]$ and satisfies $Q_j(\overline{v}) = 0$;

(b) for every p, $Q_j(p)$ strictly increases and $R_j(p)$ weakly increases as the expected number of buyers per period (the popularity of intermediary j) grows;

(c) $Q_j(p)R_j(p)$ is strictly quasiconcave on $[\underline{v}, \overline{v}]$.

Condition (a) demands that the probability of sale decreases as the seller raises the reserve price; condition (b) stipulates that an increase in popularity of an intermediary raises the likelihood of sale and does not diminish the expected revenue; condition (c) is technical and guarantees uniqueness of the optimal reserve price.

In our original model, $Q_j(p_j) = (1 - F(p_j))\eta_j$ and $R_j(p_j) = p_j$ trivially satisfy (a) – (c). It is straightforward to verify that after replacement of p_j by $R_j(p_j)$ throughout the paper, all the results will continue to hold.[12]

Let us consider a specific example. Suppose that there two intermediaries $j = 1, 2$ (auction houses) that sell products via *Vickrey auction.*[13] In every period t a seller who

[12]The second part of Lemma 1 that provides an implicit formula for the solution of decision problem (2) will depend on specific Q_j and R_j.

[13]Vickrey auction is an approximation of real proxi-bidding that is used in internet auctions.

have chosen auction house j for listing his object announces reserve price p_j, and then a Vickrey auction is run among a random sample of n_j bidders drawn from population $\mathcal{N}$.[14] As a result of the auction, the object is transferred to a winner (the highest bidder), who pays to the seller the price equal to the second highest bid (or the reserve price), or the object is returned to the seller, if no bid is above the reserve price. Regardless of the auction outcome, the seller pays to auction house j the listing fee, c_j, and, in addition, if the object is sold, the closing fee, fraction μ_j of the closing price. If the object is sold, the game ends, otherwise it proceeds to the next period.

We assume that in auction house $j = 1, 2$, in each period a new sample of n_j bidders is randomly drawn to participate in the auction. The number of bidders n_j is fixed and commonly known.[15] The number of bidders n_j that arrive to auction house j in every period is the measure of popularity of the auction house: a more popular auction house attracts more bidders.[16]

Let us calculate the seller's expected revenue from auction house j. For every reserve price p_j, the probability that the object is sold, $Q_j(p_j)$, is equal to the probability that at least one bidder has the use value above p_j, thus

$$Q_j(p_j) = 1 - F^{n_j}(p_j).$$

[14] This assumption, together with our objective to maximize the profit of intermediaries rather than sellers, differentiates this model from the broad literature on auctions with resale (Horstmann and LaCasse, 1997; Gupta and Lebrun, 1999; Haile 1999, 2000, 2001, 2003; Zheng, 2002; Calzolari and Pavan, 2006; Garatt and Tröger, 2006; Pagnozzi, 2007). The exceptions are Haile (1999, 2001) who allows new bidders (in particular, all new bidders) to participate in a re-auction; and Bikhchandani and Huang (1989), Bose and Deltas (1999, 2007) and Calzolari and Pavan (2006) who model resale to a given secondary market where the original bidders need not participate; and Matros and Zapechelnyuk (2008) who consider a similar model with a monopoly auction intermediary.

[15] The results can be generalized to the case where the number of bidders n_j is random, drawn from the same distribution in each period.

[16] An auction house with n_i bidders arriving in every period is not directly comparable to an internet shop with rate of buyers' arrival η_j. One way to compare is to assume that one period for the auction house (the time interval between opening and closing an auction) amounts for k periods in the shop (k absolute units of time, $k > n_i$), and then to compare the expected numbers of buyers per absolute unit of time, n_i/k and η_j.

The expected revenue from sale, $R_j(p_j)$, is equal to

$$R_j(p_j) = \frac{1}{1 - F^{n_j}(p_j)} \left[(Y_{n_j}(p_j) - F^{n_j}(p_j))p_j + \int_{p_j}^{1} z dY_{n_j}(z) \right]$$

where Y_{n_j} is the distribution of the second highest bidder's value among n_j bidders,

$$Y_{n_j}(z) = F^{n_j}(z) + n_j F^{n_j - 1}(z)(1 - F(z)).$$

Assumptions (a) – (c) can be verified for these Q_j and R_j, provided F satisfies the monotonic hazard rate condition, i.e., $\frac{f(z)}{1-F(z)}$ is strictly increasing (see, e.g., Krishna 2002, Ch. 2.5).

Appendix

Proof of Lemma 1

Rewrite (1) as follows,

$$u_j(p_j, u^*) = -c_j + (1 - \mu_j) \left[Q_j(p_j) \left(p_j - \frac{\delta_s u^*}{1 - \mu_j} \right) \right] + \delta_s u^*.$$

Then, it is clear that

$$\arg\max_{p_j \in [\underline{v}, \bar{v}]} u_j(p_j, u^*) = \arg\max_{p_j \in [\underline{v}, \bar{v}]} \left[Q_j(p_j) \left(p_j - \frac{\delta_s u^*}{1 - \mu_j} \right) \right].$$

The result follows from Lemma 5 below with $z = \frac{\delta_s u^*}{1 - \mu_j}$.

Lemma 5 *For every $z \geq \underline{v}$, function $Q_j(p)(p - z)$ is strictly quasi-concave in p. A unique solution p^* of the maximization problem*

$$\max_{p \in [\underline{v}, \bar{v}]} Q_j(p)(p - z) \tag{14}$$

is a unique solution of the following:

$$p^* - \frac{1 - F(p^*)}{f(p^*)} = z, \text{ if } z \leq \bar{v} - \frac{1 - F(\bar{v})}{f(\bar{v})},$$

and

$$p^* = \bar{v}, \text{ if } z > \bar{v} - \frac{1 - F(\bar{v})}{f(\bar{v})}.$$

Proof. Since $Q_j(p) = (1 - F(p))\eta_j$, the first-order condition on (14) yields

$$\eta_j f(p) \left(\frac{1 - F(p)}{f(p)} - (p - z) \right) = 0. \tag{15}$$

By the monotonic hazard rate condition, $\frac{1-F(p)}{f(p)} - (p - z)$ is strictly decreasing in p for all z and $f(p)$ is everywhere positive, hence there exists at most one solution of (15). Next, for $p = \underline{v}$ we have $F(\underline{v}) = 0$, and thus the left-hand side of (15) is strictly positive. For $p = \overline{v}$, there are two cases. If $z > \bar{v} - \frac{1-F(\bar{v})}{f(\bar{v})}$, then the left-hand side of (15) is positive for all p in $[\underline{v}, \overline{v}]$, and hence the optimal price is $p^* = \overline{v}$. Otherwise, at $p = \overline{v}$ the left-hand side of (15) is nonpositive. Thus p^* is the unique solution of (15). Note also that the left-hand side of (15) is strictly negative for all $p > p^*$ and strictly positive for all $p < p^*$, that is, $Q_j(p)(p - z)$ is strictly quasiconcave. **End of Proof**

Proof of Lemma 2

Equation (4) is equivalent to

$$\begin{aligned} u_j^* &= \max_p \left[-c_j + (1 - \mu_j) Q_j(p) \left(p - \frac{\delta_s u_j^*}{1 - \mu_j} \right) + \delta_s u_j^* \right] \\ &= -c_j + \delta_s u_j^* + (1 - \mu_j) \max_p \left[Q_j(p) \left(p - \frac{\delta_s u_j^*}{1 - \mu_j} \right) \right]. \end{aligned} \tag{16}$$

Note that the seller lists her object in shop j only if

$$c_j < (1 - \mu_j) \max_p [Q_j(p)p]. \tag{17}$$

It means that $\mu_j < 1$. After dividing both sides by $(1 - \mu_j)$ and rearranging the terms in (16), we obtain

$$\frac{c_j + (1 - \delta_s) u_j^*}{(1 - \mu_j)} = \max_p \left[Q_j(p) \left(p - \frac{\delta_s u_j^*}{1 - \mu_j} \right) \right]. \tag{18}$$

Note that from (17) the left-hand side is smaller (greater) than the right-hand side at $u_j^* = 0$ ($u_j^* = \frac{1-\mu_j}{\delta_s}\bar{v}$). Since the left-hand side is increasing and the right-hand side is decreasing in u_j^* (recall that $Q_j(\cdot)$ is non-negative), equation (18) has a unique solution. **End of Proof**.

Proof of Theorem 1

Consider arbitrary fees of both shops $a_1 = (c_1, \mu_1)$ and $a_2 = (c_2, \mu_2)$. Suppose that $c_i > 0$ and $\bar{w}_i(a_i, a_j) > 0$, $i = 1$ or 2. Since $\bar{w}_i(a_i, a_j) > 0$, shop i attracts a positive measure of sellers, $\lambda_i^H(a_i, a_j) + \lambda_i^L(a_i, a_j) > 0$. Suppose that fee a_i is shop i's best reply to the strategy a_j, that is, for every strategy $a_i' = (c_i', \mu_i')$, $\bar{w}_i(a_i, a_j) \geq \bar{w}_i(a_i', a_j)$.

Recall that the payoff of θ-type seller, $\theta = H, L$, is given by

$$u_i^\theta = -c_i + (1-\mu_i)Q_i(p^\theta)p^\theta + (1 - Q_i(p^\theta))\delta_\theta u_i^\theta, \tag{19}$$

where p^θ from Lemma 1 is given by

$$p^\theta - \frac{1 - F(p^\theta)}{f(p^\theta)} = \frac{\delta_\theta u^\theta}{1 - \mu_i}. \tag{20}$$

The payoff of shop i conditional on interaction with θ-type seller, is given by

$$w_i^\theta = c_i + \mu_i Q_i(p^\theta)p^\theta + (1 - Q_i(p^\theta))\gamma w_i^\theta. \tag{21}$$

The sum of the two payoffs is equal to

$$W_i^\theta(p^\theta) \equiv u_i^\theta + w_i^\theta = Q_i(p^\theta)p^\theta + (1 - Q_i(p^\theta))(\delta_\theta u_i^\theta + \gamma w_i^\theta). \tag{22}$$

Note that this sum does not depend on the shop i's fees directly, only via the seller's choice of the price p^θ. We will show now that, whenever $c_i > 0$, the sum of the seller's and shop i's payoffs is strictly increasing in p in a small neighborhood of p^θ. Further, we will show that there exists a fee transformation: higher μ_i and lower c_i that shifts up the seller's price, thus increasing in the sum of the payoffs and making shop i strictly better off.

Lemma 6 *If $c_i > 0$, then for each type $\theta = H, L$ there exists a neighborhood of p^θ where $W_i^\theta(\cdot)$ is strictly increasing.*

Proof. After rearranging the terms of (22), we obtain

$$W_i^\theta(p) = Q_i(p)(p - (\delta_\theta u_i^\theta + \gamma w_i^\theta)) + \delta_\theta u_i^\theta + \gamma w_i^\theta.$$

Let p^* be the price that maximizes $W_i^\theta(p)$. By Lemma 5, it is given by

$$p^* - \frac{1 - F(p^*)}{f(p^*)} = \delta_\theta u_i^\theta + \gamma w_i^\theta, \tag{23}$$

and, furthermore, $W_i^\theta(p)$ is strictly quasi-concave. Hence $W_i^\theta(p)$ is strictly increasing in p whenever $p < p^*$.

Solving (19) for u_i^θ, we obtain

$$u_i^\theta = \frac{(1 - \mu_i)Q_i(p^\theta)p^\theta - c_i}{1 - \delta_\theta(1 - Q_i(p^\theta))}.$$

Solving (21) for w_i^θ, we get

$$w_i^\theta = \frac{\mu_i Q_i(p^\theta)p^\theta + c_i}{1 - \gamma(1 - Q_i(p^\theta))}.$$

Now, using the assumption that $\gamma \geq \delta_\theta$, we obtain

$$\begin{aligned}
\delta_\theta u_i^\theta + \gamma w_i^\theta &\geq \delta_\theta \frac{(1 - \mu_i)Q_i(p^\theta)p^\theta - c_i}{1 - \delta_\theta(1 - Q_i(p^\theta))} + \delta_\theta \frac{\mu_i Q_i(p^\theta)p^\theta + c_i}{1 - \gamma(1 - Q_i(p^\theta))} \\
&\geq \delta_\theta \frac{Q_i(p^\theta)p^\theta}{1 - \delta_\theta(1 - Q_i(p^\theta))} = \frac{\delta_\theta}{(1 - \mu_i)} \frac{(1 - \mu_i)Q_i(p^\theta)p^\theta}{1 - \delta_\theta(1 - Q_i(p^\theta))} \\
&\geq \frac{\delta_\theta}{(1 - \mu_i)} \frac{(1 - \mu_i)Q_i(p^\theta)p^\theta - c_i}{1 - \delta_\theta(1 - Q_i(p^\theta))} \equiv \frac{\delta_\theta}{(1 - \mu_i)} u_i^\theta.
\end{aligned}$$

The last inequality is due to the assumption that $c_i \geq 0$ and it is strict whenever $c > 0$. Therefore, $\delta_\theta u_i^\theta + \gamma w_i^\theta > \frac{\delta_\theta u_i^\theta}{1 - \mu_i}$ whenever $c_i > 0$.

Since $z - \frac{1 - F(z)}{f(z)}$ is strictly increasing, it follows that $p^\theta < p^*$ from (20) and (23). Hence, $W(p)$ is strictly increasing in some neighborhood of p^θ. **End of Proof**.

We continue with the proof of Theorem 1. Consider a hypothetical situation where shop i can choose different fees, a_i', only for the current period, while keeping the original fees, a_i, in the next and further periods (thus affecting only payoffs from this period, but not continuation payoffs). By the one-period deviation principle, if no choice of a_i' in the current period can lead to an increase in i's profit, neither can any change of fees in all periods.

Let μ_i' be a closing fee in a small neighborhood of μ_i, $\mu_i' > \mu_i$. Since the fees will be changed only in the current period, the continuation payoff of a θ-type seller, u_i^θ, and the continuation payoff of shop i conditional on the interaction with θ-type seller, w_i^θ, remain unchanged. By (20) and by the hazard rate assumption, the price under the new closing fee, p_θ', is strictly greater than the original price, p_θ, and by Lemma 6, $W_i^\theta(p_\theta') > W_i^\theta(p_\theta)$. Let $\varepsilon > 0$ be a small positive number such that

$$0 < \varepsilon < W_i^\theta(p_\theta') - W_i^\theta(p_\theta),\ \theta = H, L.$$

Consider fees (c_i', μ_i') such that the listing fee $0 \leq c_i' < c_i$ and the expected payoff of shop i is increased by ε in comparison with fees (c_i, μ_i). Since the total payoff increase is $W_i^\theta(p_\theta') - W_i^\theta(p_\theta)$, it follows that the seller's payoff must also increase. Note that, since only current-period fees are changed, the payoffs of shop i and the seller are linear in c_i' (it is a simple redistribution of the revenue). Hence, for every measures of H-type and L-type sellers, $\lambda_i^H(a_i, a_j)$ and $\lambda_i^L(a_i, a_j)$, the shop can obtain at least ε more revenue, which is a contradiction that the initial fees (c_i, μ_i) are optimal. **End of Proof**.

Proof of Proposition 1

Proof. *Part (i).* Suppose that $\eta_1 = \eta_2$. Then a seller of any type prefers the shop with lower closing fee. If, say, $\mu_1 > \mu_2$, then shop 2 attracts sellers of both types, H-type and L-type, and it can profitably deviate by setting a slightly higher closing fee (but still below μ_1). If $\mu_1 = \mu_2 > 0$, then the sellers are indifferent between the shops, and a shop that attracts not all sellers can profitably deviate by setting a slightly lower closing fee.

Part (ii). Suppose that $\eta_1 > \eta_2$ and $\mu_1 = 0$. Then a seller of any type strictly prefers shop 1 (even when $\mu_2 = 0$), and thus shop 1 can profitably deviate by setting a slightly higher closing fee. **End of Proof**.

Proof of Proposition 2

We need to show that if fees $(\mu_1, \mu_2) \neq (1,1)$ are such that $u_1^H = u_2^H$, then $u_1^L > u_2^L$. Denote

$$\tilde{u}_j^\theta = \frac{u_j^\theta}{1-\mu_j}, \; j = 1,2 \text{ and } \theta = H, L.$$

Then, $u_j^\theta = (1-\mu_j)\tilde{u}_j^\theta$, and thus $u_1^H = u_2^H$ implies $u_1^L > u_2^L$ if and only if

$$\frac{1-\mu_2}{1-\mu_1} = \frac{\tilde{u}_1^H}{\tilde{u}_2^H} < \frac{\tilde{u}_1^L}{\tilde{u}_2^L}. \tag{24}$$

Dividing both sides of (1) by $1-\mu_j$ and maximizing the left-hand side w.r.t. p (with $c_j = 0$), we have

$$\tilde{u}_j^\theta = \max_p \left[Q_j(p)p + (1 - Q_j(p))\delta_\theta \tilde{u}_j^\theta \right].$$

It follows from Lemma 2 that the above equation has a unique solution $\tilde{u}_j^\delta$. Note that $\tilde{u}_j^\theta$ does not depend on μ_j, and neither does the optimal price p_j^θ. Also, since $\eta_1 > \eta_2$, we have $Q_1(p) > Q_2(p)$ and it is straightforward to show that $\tilde{u}_1^\theta > \tilde{u}_2^\theta$ and $p_1^\theta > p_2^\theta$, $\theta = H, L$.

Define

$$Q(\eta_j, z) = \max_p \left[Q_j(p)p + (1 - Q_j(p))z \right]. \tag{25}$$

Thus we have $\tilde{u}_j^\theta = Q(\eta_j, \delta_\theta \tilde{u}_j^\theta)$. Note that function $Q(\eta_j, z)$ satisfies the submodularity condition in (η_j, z):

$$\frac{Q(\eta_1, z_1)}{Q(\eta_2, z_1)} < \frac{Q(\eta_1, z_2)}{Q(\eta_2, z_2)}$$

whenever $\eta_1 > \eta_2$ and $z_1 > z_2$. To see this, take the partial derivative of Q with respect to z. By the Envelope Theorem, it is equal to $1 - Q_j(p)$, where p is the maximizer of (25). Since $Q_j(p)$ is strictly increasing in η_j (see Proof of Lemma 5), the submodularity of Q is immediate. Let $z_1 = \delta_H \tilde{u}_1^H$ and $z_2 = \delta_L \tilde{u}_2^L$. Using $\tilde{u}_1^H > \tilde{u}_1^L > \tilde{u}_2^L$ and $\tilde{u}_1^H > \tilde{u}_2^H > \tilde{u}_2^L$, we obtain inequality (24). **End of Proof**.

Proof of Lemma 4

Solving (19) for u_i^θ (with $c_i = 0$), we obtain

$$u_i^\theta(\mu_i) = \frac{(1-\mu_i)Q_i(p^\theta)p^\theta}{1-\delta_\theta(1-Q_i(p^\theta))}.$$

After dividing both sides by $1-\mu_i$, we have

$$\tilde{u}_j^\theta \equiv \frac{u_i^\theta(\mu_i)}{1-\mu_i} = \frac{Q_i(p^\theta)p^\theta}{1-\delta_\theta(1-Q_i(p^\theta))}.$$

By the argument provided in the Proof of Proposition 2, we know that $\tilde{u}_j$ and p^θ are independent from μ_i, and hence $u_i^\theta(\mu_i) = (1-\mu_i)\tilde{u}_j^\theta$ is linear in μ_i. Also, solving (21) for w_i^θ, we obtain

$$w_i^\theta(\mu_i) = \frac{\mu_i Q_i(p^\theta)p^\theta}{1-\gamma(1-Q_i(p^\theta))},$$

i.e., $w_i^\theta(\mu_i) = \mu_i \tilde{w}_i^\theta$, where $\tilde{w}_i^\theta$ is a constant w.r.t. μ_i. Thus,

$$w_i^\theta(\mu_i) \cdot G(u_i^\theta(\mu_i)) = \mu_i \tilde{w}_i^\theta \cdot G((1-\mu_i)\tilde{u}_j^\theta).$$

Taking the derivative with respect to μ_i and denoting $z = (1-\mu_i)\tilde{u}_j^\theta$, we obtain

$$\begin{aligned}\tilde{w}_i^\theta \cdot G(z) - \mu_i \tilde{w}_i^\theta \cdot \tilde{u}_j^\theta g(z) &= \tilde{w}_i^\theta \left(G(z) - \mu_i \tilde{u}_j^\theta g(z)\right) \\ &= \tilde{w}_i^\theta \left(G(z) - (\tilde{u}_j^\theta - z)g(z)\right).\end{aligned}$$

Since by assumption on G (see Section 2) expression $[G(z) - (\bar{v} - z)g(z)]$ is strictly increasing in z, its derivative satisfies

$$2g(z) - (\bar{v} - z)g'(z) > 0$$

and, furthermore, this inequality holds for every $z \leq \tilde{u}_j^\theta$ even after we replace $\bar{v}$ by $\tilde{u}_j^\theta$ (as $\tilde{u}_j^\theta \leq \bar{v}$). Thus $G(z) - (\tilde{u}_j^\theta - z)g(z)$ is also strictly increasing in $z \leq \tilde{u}_j^\theta$. Since z is strictly decreasing in μ_i, it follows that $\mu_i \tilde{w}_i^\theta \cdot G((1-\mu_i)\tilde{u}_j^\theta)$ is strictly concave.

Proof of Proposition 3

Proof. Condition (10) means that H-type sellers prefer shop 1 when $\mu_2 = 0$. From Proposition 2, $\phi_H(0) < \phi_L(0)$. Thus, condition (10) implies that both types of sellers prefer shop 1 when $\mu_2 = 0$ (and even more so at any higher closing fee of shop 2), and hence, setting the monopoly closing fee is the best reply for shop 1.

Conversely, if a monopoly equilibrium exists, then after setting the monopoly closing fee, μ_1^M, shop 1 attracts sellers of all types for any closing fee of shop 2, including $\mu_2 = 0$. Therefore, $\mu_1^M \leq \phi_H(0)$. **End of Proof.**

Proof of Proposition 4

Proof. Suppose that shops' closing fees are $(\phi_H(0), 0)$. Condition (12) means that H-type sellers prefer shop 2 under the monopoly fee μ_1^M, hence this is not the monopoly equilibrium. Condition (13) mean that shop 1 has no incentive to increase its closing fee to the level that would maximize the revenue from L-type sellers only (completely ignoring H-type sellers); clearly, shop 1 cannot benefit by a reduction μ_1, and shop 2 cannot benefit by an increase of μ_2.

Conversely, suppose a contestable monopoly equilibrium exists. In such an equilibrium, to attract sellers of all types, shop 1 sets its closing fee at most $\phi_H(0)$, and (12) must hold, otherwise shop 1 could have benefited by setting closing fee μ_1^M and obtaining the monopoly equilibrium profit. Similarly, (13) must hold, otherwise shop 1 could have benefited by setting closing fee μ_1^L. **End of Proof.**

Proof of Proposition 5

Proof. To prove part (i), we need to show that the reverse segmentation, where L-type sellers prefer shop 2 and H-type sellers prefer shop 1, cannot occur in equilibrium. This is immediate by Proposition 2, according to which, for every pair of fees, H-type sellers prefer shop 1 only if L-type sellers also prefer shop 1.

Part (ii). Consider, say, shop 1. By part (i), the best-reply fee is an interior solution of the problem of finding the best fee facing the population of L-type sellers only. But this solution is equal to the unique closing fee on the monopoly market with only L-type sellers, μ_1^L. **End of Proof**.

Proof of Theorem 2

Equilibria in the case of $\eta_1 = \eta_2$ are fully characterized by Proposition 1.

Suppose that $\eta_1 > \eta_2$. We described three types of equilibria, monopoly, contestable monopoly, and market segmentation equilibrium and showed that if an equilibrium exists, it is unique. It remains to show that no other equilibria may exist.

Consider an equilibrium $s = ((\mu_1, \mu_2), (\lambda_1^H, \lambda_2^H, p_1^H, p_2^H), (\lambda_1^L, \lambda_2^L, p_1^L, p_2^L))$. First, assume that $\lambda_j^\theta \in \{0, 1\}$ for every $j = 1, 2$ and every $\theta = H, L$. Note that s is a monopoly or contestable monopoly equilibrium if $\lambda_2^H = \lambda_2^L = 0$. Since $\eta_1 > \eta_2$, clearly, $\lambda_1^H = \lambda_1^L = 0$ cannot occur in equilibrium, as shop 1 can charge low enough closing fee to attract sellers (see Figure 1). Next, note that if $\lambda_1^H = 0$ and $\lambda_2^L = 0$, s is a market segmentation equilibrium, and $\lambda_2^H = 0$ and $\lambda_1^L = 0$ cannot occur in equilibrium by Proposition 2.

Finally, suppose that $0 < \lambda_j^\theta < 1$ for some $j = 1, 2$ and some $\theta = H, L$, that is, a θ-type seller is indifferent between two shops. Then s cannot be an equilibrium: by Proposition 1 (ii), at least one shop receives positive profit and thus it can attract θ-type sellers with probability one by marginally reducing its closing fee. **End of Proof.**

References

Baye, M. R. and J. Morgan (2001). "Information Gatekeepers on the Internet and the Competitiveness of Homogeneous Product Markets." *American Economic Review* **91**, 454–474.

Bikhchandani, S. and C-f. Huang (1989). "Auctions with Resale Markets: An Exploratory Model of Treasury Bill Markets." *Review of Financial Studies* **2**, 311–339.

Bose, S. and G. Deltas (1999). "Welcoming the Middlemen: Restricting Competition in Auctions by Excluding the Consumers." In *Current Trends in Economics: Theory and Applications*, Studies in Economic Theory, vol. 8 (Aliprantis, Alkan, and Yannelis, Eds.), 119–131. Springer-Verlag.

Bose, S. and G. Deltas (2007). "Exclusive vs. Non-Exclusive Dealing in Auctions with Resale." *Economic Theory* **31**, 1–17.

Brown, J. and J. Morgan (2009). "How Much Is a Dollar Worth? Tipping versus Equilibrium Coexistence on Competing Online Auction Sites." *Journal of Political Economy* **117**, 668–700.

Calzolari, G. and A. Pavan (2006). "Monopoly with Resale." *RAND Journal of Economics* **37**, 362–375.

Ellison, G.,D. Fudenberg and M. Möbius (2004). "Competing Auctions." *Journal of the European Economic Association* **2**, 30–66.

Garratt, R. and T. Tröger (2006). "Speculation in Standard Auctions with Resale." *Econometrica* **74**, 753–769.

Goldmanis, M., A. Hortaçsu, C. Syverson and Ö. Emre (2010). "E-Commerce and the Market Structure of Retail Industries." *Economic Journal* **120**, 651–682.

Gupta, M. and B. Lebrun (1999). "First price auctions with resale." *Economics Letters* **64**, 181–185.

Haile, P. (1999). "Auctions with Resale." Working paper, University of Wisconsin-Madison.

Haile, P. (2000). "Partial pooling at the reserve price in auctions with resale opportunities." *Games and Economic Behavior* **33**, 231–248.

Haile, P. (2001). "Auctions with Resale Markets: An Application to U.S. Forest Service Timber Sales." *American Economic Review* **91**, 399-427.

Haile, P. (2003). "Auctions with private uncertainty and resale opportunities." *Journal of Economic Theory* **108**, 72–110.

Horstmann, I. and C. LaCasse (1997). "Secret Reserve Prices in a Bidding Model with a Resale Option." *American Economic Review* **87**, 663-684.

Krishna, V. (2002). *Auction Theory.* Academic Press.

Matros, A. and A. Zapechelnyuk (2008). Optimal Fees in Internet Auctions. *Review of Economic Design* **12**, 155–163.

Moldovanu, B., A. Sela and X. Shi (2008). "Competing Auctions with Endogenous Quantities." *Journal of Economic Theory* **141**, 1–27.

Myerson, R. B. and M. A. Satterthwaite (1983). "Efficient Mechanisms for Bilateral Trading." *Journal of Economic Theory* **29**, 265–281.

Pagnozzi, M. (2007). "Bidding to Lose? Auctions with Resale." *RAND Journal of Economics* **38**, 1090–1112.

Shaked, A. and J. Sutton (1982). "Relaxing Price Competition Through Product Differentiation." *Review of Economic Studies* **49**, 3–13.

Singh, N. and X. Vives (1984). "Price and Quantity Competition in a Differentiated Duopoly." *RAND Journal of Economics* **15**, 546–554.

Stahl, D. O. (1989). "Oligopolistic Pricing with Sequential Consumer Search." *American Economic Review* **79**, 700–712.

Stahl, D. O. (1996). "Oligopolistic Pricing with Sequential Consumer Search and Heterogeneous Search Costs." *International Journal of Industrial Organization* **14**, 243–268.

Zheng, C. (2002). "Optimal Auction with Resale." *Econometrica* **70**, 2197–2224.

Queen Mary
University of London

This working paper has been produced by
the School of Economics and Finance at
Queen Mary, University of London

School of Economics and Finance
Queen Mary, University of London
Mile End Road
London E1 4NS
Tel: +44 (0)20 7882 5096
Fax: +44 (0)20 8983 3580
Web: www.econ.qmul.ac.uk/papers/wp.htm