## School of Economics and Finance

Separating Bayesian Updating from Non-Probabilistic Reasoning: An Experimental Investigation

Dan Levin, James Peck and Asen Ivanov



# Separating Bayesian Updating from Non-Probabilistic Reasoning: An Experimental Investigation 

By Dan Levin, James Peck, and Asen Ivanov*


#### Abstract

Through a series of decision tasks involving colored cards, we provide separate measures of Bayesian updating and non-probabilistic reasoning skills. We apply these measures to (and are the first to study) a common-value Dutch auction. This format is more salient than the strategically equivalent first-price auction and silent Dutch formats in hinting that one should condition one's estimate of the value on having the highest bid. Both Bayesian updating skills and non-probabilistic reasoning skills are shown to help subjects correct for the winner's curse, as does the saliency of the active-clock Dutch format.


JEL classification codes: D4, D7, D8

Keywords: Bayesian updating, non-probabilistic reasoning, Dutch auction

[^0]There is mounting experimental evidence that decision makers (DMs) fail miserably when facing tasks that involve making inferences. Examples include the robust and often replicated finding of systematic overbidding in first-price and second-price common-value auction experiments, and overbidding in lemons market decision tasks such as the "acquiring a company game" (sometimes called the "takeover game"). Our goal is to better understand behavior in these contexts, by focusing on how cognitive limitations affect two important drivers of behavior: Bayesian updating (BU) and non-probabilistic reasoning (NPR). In this paper, we provide evidence, based on our experimental study, that both types of skills, BU and NPR, significantly affect behavior in common value auctions. In particular, both the auction format's requirements for these skills and the deficiency of these skills at the individual level can explain overbidding that results in the winner's curse.

Violations of BU can be context-specific, such as misreading the symmetries of a problem and arriving upon the wrong probability. In these violations of Bayes' rule, the DM understands which events should be conditioned upon, but has difficulty performing the Bayesian computation accurately. An entirely distinct source of failure in making inferences does not directly involve probabilities. This can involve posing the wrong question-the DM fails to condition upon the right observations, due to a failure of insight or recognition, which would involve logical reasoning without requiring any explicit updating of probabilities. ${ }^{1}$ Both of these cognitive limitations, of BU and of NPR, may be at work together. We adopt a multi-prong approach to separately measure BU failures and NPR failures, then demonstrating the importance of each.

Both NPR and BU skills are helpful in correcting for the winner's curse. NPR here involves recognizing the adverse selection in the event of winning. This would lead the bidder to ask "what is the expected value of the object, conditional on my signal being the highest (or one of the highest)?," rather than conditional on the bidder's signal alone. Posing this question does not involve probabilities or computations. However, not conditioning on having one of the highest signals leads to overbidding. ${ }^{2}$ Now

[^1]suppose our subject has strong NPR skills and attempts to find the expected value, conditional on having the highest signal. BU involves using probabilistic reasoning (either a computation or good intuition) to evaluate the benchmark value one has posed. Given our auction environment, a subject with NPR skills but without BU skills is unlikely to realize how large a shading rate is required, and is likely to overbid in the auction. ${ }^{3}$

We study the common-value Dutch and first-price auctions, and the extent to which experimental subjects correct for the winner's curse. Our three main treatments are: (1) the "active clock" Dutch auction (AD), in which the clock price ticks down until one of the subjects "clicks" and stops the clock, paying the corresponding clock price, (2) the "silent clock" Dutch auction (SD), in which the clock ticks down until each subject stops the clock, without feedback information about whether other subjects have clicked, and (3) the sealed-bid first-price auction (FP). This is the first experimental study of the common-value Dutch auction, as far as we are aware. The games underlying these three treatments are strategically equivalent, and as such share the same behavioral predictions under Nash equilibrium and the leading behavioral theories (e.g., cursed equilibrium, level-k, and quantal response equilibrium). ${ }^{4}$ However, actual behavior may differ across the three treatments, because the treatments may require different amounts of NPR to be used by the subjects. With the AD format, a subject who contemplates stopping the clock knows that no one else has stopped the clock, making it obvious that her bid would be the highest. This hint helps her to recognize that it is likely that other subjects' signals are lower than hers. This ought to induce her to shade her bid below her signal, which helps avoid, or at least mitigate, the winner's curse effect. With the SD or the FP format, there is an additional step of NPR required for the subject to recognize that she should condition her bid on the other signals being low enough that she is the high bidder. Thus, the SD and FP formats may require significantly more NPR than the AD format, and we
could decide to bid more than her signal.
${ }^{3}$ In our design, when the common value is $v$, signals are uniformly distributed between $v-\varepsilon$ and $v+\varepsilon$. With a signal of $x$, the shading rate is defined to be $(x-b i d) / \varepsilon$. See details in the Experimental Design section. For $\varepsilon=24, v$ is between $x-24$ and $x+24$, but the expected value conditional on the high signal is approximately $x-17$, which is less than what someone without BU skills would guess.
${ }^{4}$ The behavioral predictions are the same if the degree of cursedness, distribution of levels in the level-k model, and logit function in the QRE model do not depend on the treatment.
would expect to see lower shading rates (higher bids) and more winner's curse.
The first prong of our approach is to investigate this treatment effect. We find that the AD format exhibits significantly greater shading rates and thus significantly less winner's curse than either of the other two formats requiring more NPR. Our regression results on the shading rate of the highest bidder confirm that there is a large and highly significant treatment effect, where the shading rate in the SD and FP treatments is lower than in the AD treatment. Indeed, behavior in the SD format is almost identical to behavior in the FP format, in spite of the different framing (stopping a clock vs. typing a bid). This control helps establish that the results are not due to the dynamic framing of Dutch auctions. There is a clear tendency for shading rates to rise over time, reflecting learning, although they fall far short of the BNE benchmark.

To provide an additional control that our treatment effects are not due to the dynamic framing of Dutch auctions, we also ran treatments with affiliated private value versions of the active Dutch, silent Dutch, and first-price auctions. A subject's private value is clearly the relevant benchmark value, as there is no winner's curse effect to first recognize and then quantitatively account for. Thus, the role for NPR and BU is greatly reduced. In both the common-value and private-value Dutch auctions, a subject must decide how much to shade her bid below the relevant benchmark value, in order to leave room for profits. However, the connection between NPR and BU and the auction format ( $\mathrm{AD}, \mathrm{SD}$, or FP ) should be absent with private values, and our empirical results strongly support this view.

The second prong of our approach is to investigate the role of individual differences in NPR skills and BU skills on shading rates, with the help of individual decision tasks (questions Q1-Q4, described in section II) performed before the auctions. From behavior on Q1 and Q3, we construct dummy variables indicating BU skill, and we use errors on Q2 and Q4 to indicate a lack of NPR skill. We then include these measures of BU and NPR in our shading rate regressions. For the common-value treatments, these variables are significant, both statistically and in terms of their magnitudes. The regressions control for general intelligence, as measured by SAT/ACT scores received from the University registrars. The relationship between the BU-skill dummy and shading rates provides evidence that a reduced ability to perform BU contributes
to the winner's curse in auctions. The relationship between errors on Q2 and Q4 and shading rates suggests that a reduced ability to perform NPR contributes to the winner's curse, complementing our earlier result that increased requirements for NPR (in the SD and FP auctions, vs. the AD auction) also contribute to the winner's curse. As further evidence, there is the dog that did not bark: decision task variables have no impact on shading rates in the private value auctions, where there is no winner's curse to sort out.

## I Literature Review

To put our experimental results in context, consider the leading behavioral theories. The notions of cursed equilibrium, level-k beliefs, and analogy-based expectations ${ }^{5}$ relax the Bayes Nash equilibrium (BNE) requirement that a player's beliefs about the strategies of the other players are correct, but maintain individual rationality by assuming that the player best responds to those (possibly) inconsistent beliefs. In contrast, the notion of quantal response equilibrium insists on consistent beliefs but relaxes BNE by allowing players to "tremble"-not using best responses-when implementing their strategies. ${ }^{6}$

Ivanov, Levin, and Peck (2009, 2013-henceforth ILP) consider endogenous-timing herding games, in which changing certain parameters across treatments is predicted to have a large impact on behavior according to BNE, level-k, cursed equilibrium, and to a lesser extent, QRE. On the other hand, changing these parameters does not change the nature of the NPR subjects must acquire in order to formulate strategies. ILP find highly heterogeneous behavior across subjects within a treatment, which they interpret as due to heterogeneity in cognitive abilities. ILP find that behavior is essentially identical across treatments.

The current paper takes an opposite and in some sense more powerful approach than the approach taken in ILP. We show here that behavior depends strongly on aspects of the game that influence the cognitive demands on subjects, but which

[^2]nonetheless are predicted to have no effect whatsoever on behavior according to BNE, level-k, cursed equilibrium, and QRE. We also show that lack of BU and NPR, as measured by decision task errors, lead to significantly greater susceptibility to the winner's curse.

Esponda and Vespa (2014) address issues very similar to the ones we are studying, in the context of strategic voting with common values. A ball, either red or blue, is drawn from an urn, and a subject must vote. The other two voters are computers who observe the selected ball. If the ball is red, then both computers vote "red"; if the ball is blue, then each computer independently votes "red" with a certain probability and "blue" with a certain probability. The subject is paid if the majority vote matches the selected ball. When voting is simultaneous, subjects must perform hypothetical thinking (realize to condition on being pivotal) and information extraction (realize that their vote only matters when the ball is blue). Esponda and Vespa are cleverly able to separate hypothetical thinking from information extraction with a sequentialmove treatment in which the subject knows that she is pivotal. Hypothetical thinking to condition on being pivotal is very similar to our NPR to condition on being the winning bidder. Information extraction is a different type of NPR that requires logic but no BU skill. Also, Esponda and Vespa do not have their subjects perform other decision tasks, such as our card tasks designed to independently measure BU skills and NPR skills. See also Philippos (2014) and Koch and Penczynski (2015).

For the experimental literature on private-value first-price and Dutch auctions, see Cox, Smith, and Walker (1983). They find that subjects overbid relative to the (risk-neutral) BNE, and that there is less overbidding in the Dutch auction than in the first-price auction. Cox, Smith, and Walker (1983) also explore and reject the possibility that subjects bid less aggressively in the Dutch auction because of a "utility of suspense" that induces them to wait longer before stopping the clock. Their remaining explanation is that subjects violate Bayes' rule, but they do not distinguish between this explanation and other forms of bounded rationality. Turocy, Watson, and Battalio (2007) introduce the silent-clock format, and again find that subjects bid less aggressively in the independent private values Dutch than in the sealed-bid first-price auction, with the silent-clock auction falling in the middle. Although they do not perform decision tasks meant to identify the behavioral issues, they conjecture
that the clock helps subjects recognize the tradeoff between the probability of winning and the amount won. We find no treatment differences in our affiliated private value auctions, which is the appropriate comparison to our common value framework. ${ }^{7}$

Garvin and Kagel (1993) examine common value first-price auction data generated by inexperienced bidders from earlier studies, and for parameter values analogous to ours, average profits are -6.29 . Under common values, our average profits are -7.04 in the silent-clock and -5.74 in the first-price sealed-bid, which are similar. In contrast, our average profits in the Dutch auction are considerably larger (that is, less negative), -2.38 , which we attribute to the active clock format's hint that other subjects' signals are likely to be below one's own signal.

Kagel, Harstad, and Levin (1987) previously recognized the important role of the game format in reducing the need for NPR. They find bidding above values in a second-price sealed-bid affiliated-private-value auction despite the fact that bidding one's value is (weakly) dominant. Overbidding is, however, almost entirely eliminated in the strategically equivalent English auction. A plausible explanation, offered by the authors, is that optimal behavior in the sealed-bid auction requires nontrivial reasoning through possible orders of one's bid, one's signal, and the highest bid among others. The English auction, on the other hand, eliminates the need for such NPR: as the clock-price ascends, simply answering correctly the question "should I stay or drop out?" leads to the dominant strategy.

Charness and Levin (2009) find that the winner's curse is alive and well in an individual-choice variant of the "Acquiring a Company" game. The authors attribute this to a failure of contingent reasoning: subjects fail to condition on winning when computing expected payoffs. Charness, Levin, and Schmeidler (2013) find a winner's curse in common-value auctions where all "signals" are identical and public, but may be interpreted differently by different subjects. Interestingly, no distributions are provided and there is no formal way to perform Bayesian updating. By soliciting subjects' estimates of the value, they show that subjects do not condition on hav-

[^3]ing the highest assessment of the value; indeed, many subjects bid more than their estimate.

## II Experimental Design

We conducted three common-value auction treatments and three private-value auction treatments: $F P, A D, S D, F P^{P V}, A D^{P V}$, and $S D^{P V}$. Each treatment consisted of two parts. The first part was identical in all treatments and was based on individualchoice card tasks; in the second part of each treatment, subjects bid in groups of six for a fictitious object in multiple periods of the corresponding auction. We first describe each auction format in detail. Then, we describe the card tasks. Finally, we turn to the detailed experimental procedures.

In the common-value treatments, subjects bid for a fictitious object in each auction format with (pure) common values, as in Kagel and Levin (1986). That is, the highest bidder wins the object for a profit equal to the value of the item minus her bid (all other bidders receive 0 profit). The value of the object, $V$, is uniformly distributed between $\underline{x}=25$ and $\bar{x}=225$, and given a realization, $v$, each subject receives a signal, $x$, that is independently and uniformly distributed between $v-\varepsilon$ and $v+\varepsilon$, where $\varepsilon=24$. There are $N=6$ bidders per auction. All amounts are denominated in Experimental Currency Units (ECU).

## A $F P$ Auctions

Each bidder enters a bid on her screen, and the highest bidder wins the object for a profit equal to the value of the item $v$ minus her bid (all other bidders receive 0 profit). Ties are broken randomly. The BNE bidding function for signals $x \in[\underline{x}+\varepsilon, \bar{x}-\varepsilon]$, which is given in Kagel and Levin (1986), is: ${ }^{8}$

$$
\begin{equation*}
b(x)=x-\varepsilon+\frac{2 \varepsilon}{N+1} \exp \left[-\frac{N}{2 \varepsilon}(x-(\underline{x}+\varepsilon))\right], \tag{1}
\end{equation*}
$$

The third term in the sum quickly becomes small as $x$ moves beyond $\underline{x}+\varepsilon$. In

[^4]our data analysis, we will make extensive use of shading rates in the lab and in BNE. For a subject with signal $x$ and bid $b$, her shading rate is defined as $\frac{x-b}{\varepsilon}$. In BNE, the shading rate for $x \in[\underline{x}+\varepsilon, \bar{x}-\varepsilon]$ converges rapidly towards 1 from below as $x$ moves beyond $\underline{x}$.

## B $A D$ Auctions

In the $A D$ auctions, the clock price ticks down until one of the subjects stops the clock and pays the corresponding clock price. The clock starts at a price of 250 ECU and ticks down at a speed of 1 ECU per half-second. All parameters are as in the $F P$, except that $\varepsilon=12$ in one session. The $A D$ is strategically equivalent to the $F P$ and has the same BNE.

Note that, in the FP auction, knowing to condition one's expected value of the object on being the highest bidder requires a considerable amount of NPR. On the other hand, in the $A D$ this NPR is facilitated-once a subject stops the clock, it is much more obvious that she is the highest bidder, and the salience of winning makes it easier to recognize that the expected value of the object might have to be adjusted.

## C $S D$ Auctions

The $S D$ auction is identical to the $A D$, except that the clock ticks down until each subject stops the clock, without knowing whether or not she is the high bidder until afterwards. The $S D$ is strategically equivalent to the $F P$ and $A D$, and has the same BNE. Note that in the $S D$ stopping the clock does not imply that one is the highest bidder. Thus, as in the $F P$, recognizing to condition one's expected value of the object on being the highest bidder in the $S D$ auction requires a significant amount of NPR.

The rationale for including the $S D$ in our experiment is the following. It is possible that the $F P$ and the $A D$ differ not only in the need for NPR but also in other theoretically irrelevant but psychologically important aspects as well. For example, it is possible that the sense of excitement or mode of reasoning are different in these two auction formats. Because of this, we cannot unequivocally attribute differences in behavior between the $F P$ and the $A D$ to the different need for NPR in the two
formats. The idea behind the $S D$ is that, by sharing the clock format with the $A D$, it serves as a control that virtually eliminates any differences between the $F P$ and the $A D$, other than the different need for NPR. ${ }^{9}$

## D Private Value Auctions

The affiliated private-value auction treatments are exactly like their common-value counterparts, except for the way payoffs are calculated. The processes for $v$ and $x$ are the same as before, but now x is the subject's private value. The highest bidder, with a bid of $b$, wins the object for a profit equal to $x-b$ (all other bidders receive 0 profit). The BNE bidding function for signals $x \in[\underline{x}+\varepsilon, \bar{x}-\varepsilon]$, which is given in Kagel, Harstad, and Levin (1987), is

$$
\begin{equation*}
b(x)=x-\frac{2 \varepsilon}{N}+\frac{2 \varepsilon}{N(N+1)} \exp \left[-\frac{N}{2 \varepsilon}(x-(\underline{x}+\varepsilon))\right], \tag{2}
\end{equation*}
$$

where the last term in the sum quickly becomes small as $x$ moves beyond $\underline{x}+\varepsilon$.

## E Card Tasks

Consider a deck containing two cards: one of the cards is black on both sides, and the other card is black on one side and white on the other side. One of the two cards is randomly selected and one of its sides is randomly selected to be face up, where by "randomly" we mean with equal probability. In our first decision task (Q1), the subject observes the face-up side and is asked to assess the probability that the face-down side is black. In our second decision task (Q2), the subject is asked: if a mathematically sophisticated expert will win 60 euros by correctly guessing the color of the face-down side, what is the most she would be willing to pay in order to observe the face-up side? Tasks Q3 and Q4 ask subjects the analogous questions related to a three-card deck containing a black-black card, a black-white card, and a white-white card. That is, Q3 shows the subject the randomly drawn face-up side, and asks for

[^5]the probability that the face-down side is black. Q4 asks about willingness to pay to observe the face-up side.

To relate BU and NPR skills to behavior in the auctions at the individual level, we let subjects answer questions Q1-Q4 before starting the auctions. The exact wording of Q1-Q4 is given in the online appendix. On each question, subjects are paid 10 ECU if the answer (in percentages) is within 1 of the mathematically correct answer, 5 ECU if the answer is within 5 , and 1 ECU if the answer is within 10.

Q1 is about BU. For a subject who observes black as the face-up side (as is the case for most subjects), straightforward application of Bayes rule yields the correct answer $\operatorname{Prob}\left(\right.$ black face-down|black face-up) $=66.67$ percent ${ }^{10}$; if the subject can update in the right direction but cannot perform the exact computation, she should at least realize that the mere fact that black is up is more indicative of the black-black card and she should give an answer strictly above 50 percent. If the subject mistakenly reasons that both cards are still equally possible and gives an answer of 50 percent, this indicates a failure of BU. When the face-up side is white, it should be obvious that the facedown side must be black. Thus, the correct answer is Prob(black face-down|white face-up) $=100$ percent, and we associate answers below 100 with deficiency of BU. ${ }^{11}$

On Q2, as long as a subject realizes that black is at least as likely to be face-down as white, whatever the face-up side, she should answer 0 (i.e., not be willing to pay anything) because, regardless of whether the face-up side is black or white, guessing that the face-down side is black is an optimal guess. ${ }^{12}$ Q2 is about NPR because, to recognize that there is no value in observing the face-up card, a subject merely needs to ask herself how knowing the face-up color would affect the hypothetical decisionmaker's guess. Moreover, extremely large answers indicate a serious failure of NPR, possibly due to a mistaken view that the question is asking about the total payoff

[^6]rather than the incremental payoff (value of information), or other confusion about what the question is asking.

Q3 is about Bayesian updating. For a subject who observes black as the face-up side, straightforward application of Bayes rule yields the correct answer Prob(black face-down $\mid$ black face-up $)=66.67$ percent; if the subject can update in the right direction but cannot perform the exact computation, she should at least realize that the mere fact that black is up is more indicative of the black-black card than of the black-white card (and makes the white-white card impossible) and she should give an answer strictly above 50 percent. For a subject who observes white as the face-up side, straightforward application of Bayes rule yields the correct answer Prob(black face-down|white face-up $)=33.33$ percent; if the subject can update in the right direction but cannot perform the exact computation, she should at least realize that the mere fact that white is up is more indicative of the white-white card than of the black-white card (and makes the black-black card impossible) and she should give an answer strictly below 50 percent. ${ }^{13}$

Q4 is the most complicated. Without observing the face-up color, the hypothetical decision-maker obtains an expected payoff of 30 euro by guessing either black or white for the face-down color. If she observes the face-up color, the hypothetical decisionmaker obtains an expected payoff of 40 euro by guessing that the face-down color is the same as the face-up color. Thus, observing the face-up card yields an advantage of $40-30=10$ euro and the correct answer on Q4 is 10 . Extremely large answers of 30 or more indicate a serious failure of NPR, possibly due to a mistaken view that the question is asking about the total payoff rather than the incremental payoff (value of information), or other confusion about what the question is asking.

## F Experimental Procedures

Table 1 provides a summary of the sessions we ran. For the common-value treatments, there were $16-26$ subjects per session and we had a total of 94 subjects in

[^7]| Session | Subject <br> IDs | Treatment | Location | $\varepsilon$ | Current version <br> of card tasks? |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Session 1 | $1-21$ | $A D$ | OSU | 12 | No |
| Session 2 | $22-37$ | $S D$ | OSU | 12 | No |
| Session 3 | $38-58$ | $A D$ | OSU | 24 | No |
| Session 4 | $59-74$ | $S D$ | OSU | 24 | No |
| Session 5 | $75-98$ | $A D$ | OSU | 24 | Yes |
| Session 6 | $99-119$ | $A D$ | VCU | 24 | Yes |
| Session 7 | $120-139$ | $A D$ | OSU | 24 | Yes |
| Session 8 | $140-158$ | $A D$ | VCU | 24 | Yes |
| Session 9 | $159-178$ | $S D$ | VCU | 24 | Yes |
| Session 10 | $179-196$ | $S D$ | VCU | 24 | Yes |
| Session 11 | $197-222$ | $S D$ | OSU | 24 | Yes |
| Session 12 | $223-243$ | $S D$ | OSU | 24 | Yes |
| Session 13 | $244-263$ | $F P$ | OSU | 24 | Yes |
| Session 14 | $264-280$ | $F P$ | VCU | 24 | Yes |
| Session 15 | $281-302$ | $F P$ | VCU | 24 | Yes |
| Session 16 | $303-319$ | $F P$ | VCU | 24 | Yes |
| Session 17 | $320-337$ | $F P$ | OSU | 24 | Yes |
| Session 18 | $338-364$ | $A D^{P V}$ | OSU | 24 | Yes |
| Session 19 | $365-391$ | $A D^{P V}$ | OSU | 24 | Yes |
| Session 20 | $392-415$ | $S D^{P V}$ | OSU | 24 | Yes |
| Session 21 | $416-438$ | $S D^{P V}$ | OSU | 24 | Yes |
| Session 22 | $439-453$ | $F P^{P V}$ | OSU | 24 | Yes |
| Session 23 | $454-480$ | $F P^{P V}$ | OSU | 24 | Yes |
| Session 24 | $481-508$ | $F P^{P V}$ | OSU | 24 | Yes |

Table 1: Summary of sessions
the $F P$ treatment, 126 subjects in the $A D$ treatment, and 117 subjects in the $S D$ treatment. For the private-value treatments, there were $15-28$ subjects per session and we had a total of 70 subjects in the $F P^{P V}$ treatment, 54 subjects in the $A D^{P V}$ treatment, and 47 subjects in the $S D^{P V}$ treatment. Sessions were run either at the Experimental Economics Lab at The Ohio State University (OSU) or at the Experimental Laboratory for Economics and Business Research at Virginia Commonwealth University (VCU). Subjects were undergraduate students at OSU and VCU drawn from a wide array of majors. Subjects in the first four sessions were given a slightly different version of Q1-Q4. We changed this version in later sessions because we felt
it was too complicated. ${ }^{14}$
Each session lasted 100-120 minutes. At the start of each session, the experimenter read the instructions (see online appendix) for the card tasks aloud as subjects read along, seated at their computer terminals. After clarifying questions, the card tasks were started with Q1-Q4 successively appearing on subjects' computer screens. ${ }^{15}$

After the card tasks were completed, the experimenter read the auction instructions (see online appendix) aloud as subjects read along. In each period, subjects were randomly matched into markets of six, and a new random matching was performed in each new period. ${ }^{16}$ After clarifying questions, subjects went through 2-4 dry-run periods. ${ }^{17}$ After additional questions, the actual auctions were started. There was a total of 30 periods. ${ }^{18}$ At the end of each common-value auction, each subject was provided feedback on the market price, the resale value, the winning bidder's profit or loss, her own profit or loss, her cash balance entering the period, and her current cash balance. She could also see her own signal, the winning bidder's signal, and the signals of all buyers in her market. At the end of each private-value auction, each subject was provided feedback on the market price, the winning bidder's private value and profit or loss, her own profit or loss, her cash balance entering the period, and her current cash balance. She could also see her own private value, the winning bidder's private value, and the private values of all buyers in her market. The experiment was programmed and conducted with the software z-Tree (Fischbacher (2007)). In the online appendix, we provide screen-shots from auctions in the $F P, A D$, and $S D^{P V}$ treatments.

Because the auctions could involve losses, subjects were provided with an initial cash balance of 25 ECU in the common-value treatments and 6 ECU in the private-

[^8]value treatments. ${ }^{19}$ Any earnings from the card tasks were added to the initial cash balance. Then, profits/losses during the auctions were added to/subtracted from subjects' balances. If a subject's balance fell below 0 during the session, she was paid $\$ 5$ (common value sessions) and $\$ 7$ (private value sessions), and asked to leave. ${ }^{20}$ For subjects who did not go bankrupt by the end of the session, their ECU earnings were converted into dollars at the exchange rate of 50 cents per ECU and subjects were paid either the resulting amount or the bankruptcy payment, whichever was greater. Average earnings in the $A D / S D / F P$ treatment were $\$ 14.34 / \$ 11.71 / \$ 10.30$; average earnings in the $A D^{P V} / S D^{P V} / F P^{P V}$ treatment were $\$ 13.86 / \$ 12.66 / \$ 14.05{ }^{21}$

## III Results

In this section, we start by providing some descriptive statistics for the data from the card tasks. Then, we analyze the data from the CV auctions with an emphasis on comparing behavior between auction formats. After that, we turn to the connection between behavior in the card tasks and in the CV auctions. Then, we analyze the data from the PV sessions. Finally, we address learning by bidders across auction periods.

Before we proceed, let us make some brief remarks regarding the data used in the statistical analysis. First, any results involving the card tasks are based on data excluding sessions 1-4 (in which the card tasks were different from and, hence, not comparable to the card tasks in later sessions).

Second, as in the literature on common value auction experiments with this design, we restrict attention to observations with $25+\varepsilon \leq x \leq 225-\varepsilon$. For comparability, expected profits of the winning bidder in the risk-neutral NE are also computed conditional on $25+\varepsilon \leq x \leq 225-\varepsilon$.

[^9]Third, occasionally subjects would place a weakly dominated bid strictly above $x+\varepsilon$. Such bids are sometimes "mouse mistakes" and in any case would have an undue influence on average shading rates, so these bids are dropped from the analysis unless otherwise noted. ${ }^{22}$

Fourth, occasionally subjects would place a bid strictly below $x-2 \varepsilon$. Such bids distort our regressions relating responses on Q1-Q4 to shading rates. Higher shading rates reflect more correction for the winner's curse and should be positively correlated with decision task performance for shading rates less than 1 (bids above $x-\varepsilon$ ). This positive relationship between decision task performance and shading rates should no longer hold with shading rates above some cutoff. To correct for this distortion but minimize the amount of data dropped from the analysis, shading rates greater than 2 (bids below $x-2 \varepsilon$ ) are dropped from the analysis unless otherwise noted. ${ }^{23}$

Fifth, average shading rates, as well as any regressions with shading rates as dependent variable that use data from all three auction formats, are based solely on highest bids (i.e., bids that are at least as high as any other bid on the market). We restrict attention only to highest bids because only these bids are observable in the $A D$ treatment, and we want to maintain comparability across treatments. Regressions with shading rate as dependent variable that use only data from the $S D$ and $F P$ treatments are based on all bids (subject to the constraints mentioned in the previous points).

## A Card Tasks

Figure 1 presents histograms of subjects' responses on Q1-Q4. In the top left panel, we see that, although more than $70 \%$ of subjects observing white face-up on Q1 give the (obvious) correct response of 100 , a substantial minority give an incorrect answer. In the top right panel and the two panels on the next row, we see that around $60 \%$ of

[^10]

Figure 1: Histograms of answers on Q1-Q4.
subjects give an answer of 50 on Q1 when black is face-up and on Q3. ${ }^{24}$ In fact, only

[^11]$33 \%$ of subjects give an answer strictly above 50 when black is face-up on Q1, only $27 \%$ of subjects update in the correct direction when white is face-up on Q3 (by giving an answer strictly below 50 ), and only $10 \%$ of subjects update in the correct direction when black is face-up on Q3 (by giving an answer strictly above 50). A subject might update in the wrong direction on Q3 if she fails to eliminate from consideration the card with both sides of the color she does not observe. ${ }^{25}$.

Since it is not clear whether the naive answer of 50 is much better than an answer on the wrong side of 50 , we do not use the absolute errors on Q1 and Q3 to measure BU skills. Rather, to indicate BU skills, we define two dummy variables, $Q 1 b Q 3$ and $Q 1 w Q 3 . Q 1 b Q 3$ is defined for a subject who observed black-up on Q1. It equals 1 if that subject gave an answer strictly greater than 50 on Q1 and updated in the correct direction on Q3; it equals 0 otherwise. $Q 1 b Q 3$ equals 1 for only $9 \%$ of the eligible subjects (who observed black-up on Q1). $Q 1 w Q 3$ is defined for a subject who observed white-up on Q1. It equals 1 if that subject gave the correct response of 100 on Q1 and updated in the correct direction on Q3; it equals 0 otherwise. $Q 1 w Q 3$ equals 1 for only $7 \%$ of the eligible subjects (who observed white-up on Q1).

The bottom two panels show that there is a lot of heterogeneity in responses to Q2 and Q4. Oddly enough, a majority of subjects (56\%) give a response of at least 30 ECU on Q2, and a majority of subjects (53\%) give a response of at least 30 ECU on Q4. Notice that guessing black without any information yields our hypothetical decision maker a payoff of 45 ECU with the two-card deck and 30 ECU with the three-card deck, so the marginal benefit of perfect information is only 15 ECU on Q2 and 30 ECU on Q4! Some of the subjects could be responding with their estimate of the total profits of the hypothetical decision maker with the information, not the incremental value of the information as asked. Other subjects could simply be confused about what the question is asking. The large mistakes we observe are clearly not Bayesian updating failures, but they could represent a combination of non-probabilistic reasoning failures. Because large errors seem to be worse failures than smaller errors, and because we do not know where to draw the line, we use the absolute errors on Q2 and Q4 to indicate (lack of) NPR skills.

[^12]To measure NPR skills, we use two variables, ErrorQ2 and ErrorQ4, which equal the standardized absolute size of the error on Q2 and Q4, respectively. As a means to control for general intelligence when analyzing the connection between behavior on Q1-Q4 and in the auctions, we also obtained subjects' SAT and ACT scores from the OSU and VCU Registrar's offices. Because both scores are not available for all subjects, we computed a new variable, SAT1. For subjects whose SAT score we have, SAT1 equals their standardized SAT score; for subjects whose SAT score we do not have but whose ACT score we have, we set SAT1 equal to the standardized converted SAT score based on their ACT score. ErrorQ2, ErrorQ4, and SAT1 are all standardized (i.e., we subtract the mean and divide by the standard deviation), so that the coefficient on each of these variables in later regressions will show the effect of increasing the variable by one-standard deviation.

## B CV Auctions

We would now like to compare behavior between the three CV auction formats. We start by looking at some descriptive statistics regarding bankruptcy rates, shading rates, and profits. After that, we will proceed with more formal regression analysis.

Figure 2 shows the fraction of bankrupt subjects for periods 1-30 in each treatment. Observe that this fraction is consistently lower in the $A D$ auctions than in the $S D$ or $F P$ auctions.

Table 2 shows the average shading rates of the highest bidder in each auction, broken down by blocks of periods. In all treatments, average shading rates are well below the NE benchmark (which is approximately 1) in all periods. Pooling across periods 1-30, average shading rates are higher in the $A D$ auctions. This difference is mainly driven by higher shading rates in the $A D$ auctions in earlier periods. (We discuss learning and how it compares across treatments in section E.)

Table 3 shows, for sessions with $\varepsilon=24$, the actual average profit of the winning bidder in each treatment, broken down by blocks of periods, as well as the expected profit of the winning bidder in the risk neutral NE. ${ }^{26}$ Winning bidders are losing money in all treatments. However, they are losing less in the $A D$ auctions.

[^13]

Figure 2: Cumulative fraction of bankrupt subjects.

To recap, the summary evidence based on bankruptcies, average shading rates, and average profits is in line with the view that the $A D$ format mitigates the winner's curse by facilitating the NPR that one should condition on being the highest bidder, while the other two formats do not facilitate this reasoning.

To more formally compare behavior across treatments, we would like to estimate the following regression (using only highest bids for comparability between the three treatments):

$$
\begin{align*}
S R_{i t}=\beta_{0} & +\beta_{1} S D_{i}+\beta_{2} F P_{i}+\beta_{3} t+\beta_{4} t \times S D_{i}+\beta_{5} t \times F P_{i}+ \\
& +\beta_{6} \text { Cash }_{i t}+\beta_{7} \operatorname{Cash}_{i t}^{2}+\beta_{8} O S U_{i}+\beta_{9} E p s 24_{i}+u_{i t} \tag{3}
\end{align*}
$$

where $S R_{i t}$ is the shading rate of subject $i$ in period $t, S D_{i}$ is a dummy that equals 1 in the $S D$ treatment (allows us to check whether shading rates at the start of the session are higher in the $S D$ than in the $A D$ auctions), $F P_{i}$ is a dummy that equals 1 in the $F P$ treatment (allows us to check whether shading rates at the start of the session are higher in the $F P$ than in the $A D$ auctions), $t \times S D_{i}$ is an interaction term between $t$ and $S D_{i}$ (allows for differential learning in the $S D$ auctions), $t \times F P_{i}$

|  | $A D$ | $S D$ | $F P$ |
| :--- | :---: | :---: | :---: |
| Actual, periods 1-5 | 0.28 | 0.12 | 0.05 |
| Actual, periods 6-10 | 0.47 | 0.26 | 0.20 |
| Actual, periods 11-15 | 0.54 | 0.31 | 0.44 |
| Actual, periods 16-20 | 0.54 | 0.33 | 0.54 |
| Actual, periods 21-25 | 0.57 | 0.59 | 0.53 |
| Actual, periods 26-30 | 0.57 | 0.61 | 0.56 |
| Actual, periods 1-30 | 0.49 | 0.33 | 0.36 |
| Risk-neutral NE | $\approx 1$ | $\approx 1$ | $\approx 1$ |

Table 2: Average shading rate of highest bidder and approximate shading rate in risk-neutral NE.

|  | $A D$ | $S D$ | $F P$ |
| :--- | :---: | :---: | :---: |
| Actual, periods 1-5 | -6.09 | -10.49 | -12.17 |
| Actual, periods 6-10 | -2.97 | -9.87 | -6.92 |
| Actual, periods 11-15 | -1.59 | -8.16 | -4.79 |
| Actual, periods 16-20 | -0.34 | -4.18 | -5.41 |
| Actual, periods 21-25 | -1.06 | -1.69 | -1.80 |
| Actual, periods 26-30 | -0.81 | -2.79 | -1.17 |
| Actual, periods 1-30 | -2.38 | -7.04 | -5.74 |
| Risk-neutral NE | 6.48 | 6.48 | 6.48 |

Table 3: Actual average profit (in ECU) of winning bidder and expected profit of winning bidder in risk-neutral NE when $\varepsilon=24$.
is an interaction term between $t$ and $F P_{i}$ (allows for differential learning in the $F P$ auctions), Cash $h_{i t}$ captures $i$ 's cash balance in ECU at the start of period $t$ (allows us to control for cash balance effects on bidding behavior; including the square of this variable allows it to have a nonlinear effect on shading rates), $O S U_{i}$ is a dummy that equals 1 if $i$ participated in the sessions held at OSU, $E p s 24_{i}$ is a dummy that equals 1 if $\varepsilon=24$, and $u_{i t}$ is the error-term. (Below, we drop the " $i$ " and " $t$ " subscripts on variables.)

In this regression, Cash may be correlated with the error term. ${ }^{27}$ Therefore, we

[^14]

Table 4: Regression results for CV sessions. Standard errors shown in parentheses (clustered at the subject level in columns 1-3; heteroskedasticity robust in column 4). $* / * * / * * *$ indicates statistical significance at the $10 / 5 / 1$ percent level.
instrument Cash and Cash ${ }^{2}$ with StartCash, the cash balances with which a subject enters the auctions (i.e., initial cash balance + earnings from the card tasks), and StartCash ${ }^{2}$. We then use two-stage least squares, pooling all observations across $i$ and $t$ and clustering standard errors at the subject level. ${ }^{28}$

The first column of Table 4 reports the regression results. Looking at the estimated coefficients on $S D$ and $F P$, we see that bidding at the start of the session is very similar in the $S D$ and $F P$ formats (any difference is economically small and statistically not significant) and is much lower in the $S D$ and $F P$ formats than in the $A D$ format-by 0.294 and 0.264 , respectively (where these differences are statistically significant).

We also run the analogue of Regression (3) with the winning bidder's currentperiod profit, Profit, as dependent variable. The results are reported in the third column of Table 4. These results parallel those in the first column. Notably, the estimated coefficients on $S D$ and $F P$ show that profits at the start of the session are very similar in the $S D$ and $F P$ formats (any difference is economically small and statistically not significant) and are much lower in the $S D$ and $F P$ formats than in the $A D$ format-by 6.433 ECU and 6.342 ECU , respectively (where these estimates are statistically significant).

## C Relationship between Performance on Card Tasks and Behavior in CV Auctions

We now turn to investigating the connection between behavior in the card tasks and behavior in the CV auctions. We run a version of Regression (3) that includes $Q 1 b Q 3$, Q1wQ3, ErrorQ2, ErrorQ4, and SAT1 as additional variables. ${ }^{29}$ We also include the dummy variable BlackupQ1 that equals 1 if the subject saw black up on Q1. This dummy allows for a different intercept for subjects depending on what color they observed as face-up on Q1 and, hence, depending on how difficult Q1 was for them.

[^15]In running this regression, we restrict attention to the $S D$ and $F P$ treatments because, if we included the data from the $A D$ treatment, we would have to use only highest bids. Highest bids involve sample selection that might make our estimates of the coefficients on the card ask variables inconsistent. ${ }^{30}$

The regression results are reported in the second column of Table 4. The most important finding is that the coefficients on $Q 1 b Q 3, Q 1 w Q 3$, ErrorQ2, and ErrorQ4 are economically large and statistically significant (at the ten percent level in the case of ErrorQ4). Based on these estimates, going from $Q 1 b Q 3=0$ to $Q 1 b Q 3=1$ when black is face-up leads to an increase in shading rates by 0.294 . Increasing the errors on Q2 and Q4 by one standard deviation leads to a decrease in shading rates by 0.085 and 0.065 , respectively.

To investigate the connection between the card task variables and profits in the auctions, we compute for each subject her average profit per auction, MeanProfit. ${ }^{31}$ Then, we run a simple cross-sectional OLS regression of MeanProfit on SD,FP, the card task variables, and $S A T 1$.

The results of this regression using all subjects from all CV treatments are displayed in the last column of Table 4. In line with the results in the third column of the table, $S D$ and $F P$ are significant, albeit at the 10-percent level. $Q 1 b Q 3$ also has a large significant effect on average profits. Q1wQ3, ErrorQ2, and ErrorQ4 have the expected signs, but are not statistically significant. The statistically significant effects of $Q 1 w Q 3$, ErrorQ2, and ErrorQ4 on shading rates may not translate into statistically significant effects on average profits because winning is to some extent random and the profit/loss conditional on winning is random, which introduces additional noise.


Table 5: Regression results for PV sessions. Standard errors shown in parentheses (clustered at the subject level in columns 1-3; heteroskedasticity robust in column 4). $* / * * / * * *$ indicates statistical significance at the $10 / 5 / 1$ percent level. R-squared is not reported in column 2 because it is negative. (This is possible in the case of instrumental variables estimation.)

## D PV sessions

We reran the regressions presented in Table 4 using the data from the PV sessions. The results are presented in Table 5. As one can see from this table, there is no evidence that the auction format or card task variables have any effect whatsoever on shading rates or profits. This lack of evidence supports our argument that the effect of auction format in the common-value auctions is due to the hint provided by the active clock that other subjects are likely to have lower signals. It also supports our argument that BU and NPR skills, as measured by the card tasks, yield higher shading rates in the common-value auctions because these skills are needed to correct for the winner's curse.

## E Learning

From Table 2 we see that, as the session progresses, average shading rates for the highest bidders in the CV auctions increase over time, and this occurs more rapidly in the $F P$ and $S D$ treatments. These findings are confirmed in the regression in the first column of Table 4: the coefficients on $t, t \times S D$, and $t \times F P$ are all significant and average shading rates in the $A D / S D / P V$ treatment increase by $0.08 / 0.19 / 0.19$ every ten periods. The coefficient on $t$ in the second column of Table 4 is also significant, showing that shading rates also increase over time if we look at all bids in the $S D$ and $F P$ treatments. Analogous remarks apply to profits if we look at the evolution of average profits over time in Table 3 as well as at the coefficients on $t, t \times S D$, and $t \times F P$ in the third column of Table $4 .{ }^{32}$

Two questions arise. First, is the increase in shading rates due to learning or attrition (subjects who bid aggressively simply go bankrupt)? Second, is the faster increase in shading rates in the $S D$ and $F P$ auctions due to faster learning (say,

[^16]because subjects in these auctions make higher losses or because they observe more of their fellow-subjects go bankrupt) or due to the fact that more subjects are going bankrupt (which means that the subject pool remaining in later periods in these auctions consists of inherently more cautious subjects)? To address these questions, we reran the regressions from the first and second columns of Table 4 by restricting attention only to bidders who never go bankrupt. The coefficient on $t$ remains significant in both regressions. ${ }^{33}$ However, the coefficients on $t \times S D$ and $t \times F P$ lose their statistical significance in the first regression, indicating that the differential increase in shading rates across treatments is primarily due to the differential rates of bankruptcies.

## IV Conclusions

We find strong evidence that the active clock feature of the $A D$ treatment results in significantly larger shading rates and profits. We suggest that the saliency of a ticking clock that has not been stopped by another bidder helps subjects reason that being the first to stop the clock suggests that others have less favourable estimates of the common value. This helps them to recognize the adverse selection problem and better correct for it. We also find that having more skill at NPR, as measured by errors on $Q 2$ and $Q 4$ of the decision tasks, leads to significantly higher shading rates, even after correcting for general intelligence (as measured by SAT1). Having more BU skill, as measured by errors on the Bayesian updating decision tasks, also leads to significantly larger shading rates, even after correcting for general intelligence.

One might conjecture that NPR skill ought to have a smaller impact on shading rates in the $A D$ treatment than in the $F P$ and $S D$ treatments, since the $A D$ format provides a hint, so that NPR is less important than in the other treatments. On the other hand, perhaps the NPR required to correct for the winner's curse is extremely difficult, so that NPR skill and the hint provided by the $A D$ format complement each other to increase shading rates. Perhaps due to these conflicting forces, we do not find evidence of interaction effects between decision task errors and treatment. In other words, it is not clear who benefits more from the greater saliency of the AD

[^17]format: those with higher or lower NPR skills.
We do not offer a general behavioral theory: the precise reasoning needed in general games and decision problems, how decision makers with limited NPR behave, and so on. Thus, this work is less ambitious or general than works such as cursed equilibrium, level-k, and QRE. Yet, our contribution to the broad field of behavioral economics goes beyond the context of auctions. First, we show that behavior in Bayesian games can sometimes be separated across several dimensions. Here, the dimensions are BU skill and NPR skill. We show that measurements of these skills based on decision tasks are correlated with skillful behavior in unrelated games, even after controlling for general intelligence. Second, our results contribute to behavioral mechanism design. We have documented that behavior varies across strategically equivalent auctions with different cognitive burdens. A designer interested in reducing the winner's curse should choose the AD format, because it reduces the cognitive burden required of bidders. A designer interested in maximizing seller revenue should prefer the FP format to the AD format, unless losses and bankruptcies are seen as a problem. Understanding these biases can help us design better mechanisms or markets.

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# Separating Bayesian Updating from Non-Probabilistic Reasoning: An Experimental Investigation 

Dan Levin, James Peck, and Asen Ivanov

## Online Appendix: Instructions and Screen Shots

After being shown pictures of both sides of each of the cards, the subjects answered the following questions on their computer screens:

Situation 1. From two cards, black-black or black-white, we randomly selected one card and then randomly selected which side to be face up. You can see the face-up side of this card on your screen. Please give your best assessment of the probability that the face-down side is black, by typing a number between 0 and 100 in the box below, and clicking OK. For example, the number 40 corresponds to a $40 \%$ chance that the face-down side is black, or a probability of 0.40 , the number 60 corresponds to a $60 \%$ chance that the face-down side is black, or a probability of 0.60 , and so on.

Your earnings will be based on how close your number is to the mathematically correct percentage. If your number equals the correct percentage or is off by no more than 1, you will receive 10 ECU ; if your number is off from the correct percentage by more than 1 but is within 5 , you will receive 5 ECU; otherwise, you will receive 0 ECU.

Situation 2. Once again, from two cards, black-black or black-white, we randomly selected one card and then randomly selected which side to be face up. Suppose that a mathematically sophisticated player who currently does not see the face-up side must guess whether the face-down side is black or white, and will be paid 60 Euros if her guess is correct. We are asking you to determine the maximum amount that she would be willing to pay for the certain knowledge of the face-up side of the card. Please give your best assessment of how much the sophisticated player would value the information of the face-up side (in Euros), by typing a number between 0 and 60 in the box below, and clicking OK.

Your earnings will be based on how close your number is to the mathematically correct calculation. If your number equals the correct value of the information, or is off by no more than 1 , you will receive 10 ECU; if your number is off from the correct calculation by more than 1 but is within 5 , you will receive 5 ECU; otherwise, you will receive 0 ECU.

Situation 3. From three cards, black-black or black-white or white-white, we randomly selected one card and then randomly selected which side to be face up. You can see the face-up side of this card on your screen. Please give your best assessment of the probability that the face-down side is black, by typing a number between 0 and 100 in the box below, and clicking OK. For example, the number 40 corresponds to a $40 \%$ chance that the face-down side is black, or a probability of 0.40 , the number 60 corresponds to a $60 \%$ chance that the face-down side is black, or a probability of 0.60 , and so on.

Your earnings will be based on how close your number is to the mathematically correct percentage. If your number equals the correct percentage or is off by no more than 1 , you will receive 10 ECU ; if your number is off from the correct percentage by more than 1 but is within 5 , you will receive 5 ECU; otherwise, you will receive 0 ECU.

Situation 4. Once again, from three cards, black-black or black-white or white-white, we randomly selected one card and then randomly selected which side to be face up. Suppose that a mathematically sophisticated player who currently does not see the face-up side must guess whether the face-down side is black or white, and will be paid 60 Euros if her guess is correct. We are asking you to determine the maximum amount that she would be willing to pay for the certain knowledge of the face-up side of the card. Please give your best assessment of how much the sophisticated player would value the information of the face-up side (in Euros), by typing a number between 0 and 60 in the box below, and clicking OK.

Your earnings will be based on how close your number is to the mathematically correct calculation. If your number equals the correct value of the information, or is off by no more than 1 , you will receive 10 ECU; if your number is off from the correct calculation by more than 1 but is within 5 , you will receive 5 ECU; otherwise, you will receive 0 ECU.

## Auction Instructions for $A D$ Treatment

1. We will now create a market in which you will act as buyers of a fictitious commodity in a sequence of trading periods. A single unit of the commodity will be auctioned off in each trading period. There will be several trading periods.
2. At the start of each period, participants are randomly placed into "markets" containing six (6) participants. If the number of available participants is not an exact multiple of six, there will be a small number of participants that are not placed in a market and who will sit out until the next period.

The random assignment of participants to markets will be done separately and independently for each period. This means that the chances that you and any given other participant are in the same market in the current period do not depend on whether the two of you were in the same market in previous periods. Rarely, this randomization procedure can have the same participant sit out for two or more consecutive periods.
3. Your task is to bid for the commodity in competition with the other five buyers in your market. In each period, the computer will select a RESALE VALUE for the unit being auctioned. This indicates the value to you of purchasing the unit in your market. This value may be thought of as the amount you would receive if you were to resell the unit. The process of determining resale values and your signals regarding resale values will be described in Sections 6 and 7 below.
4. In the middle of the screen, you will see a rectangular clock displaying a price in ECUs, which we refer to as the "Clock Price." In each new period, the market will begin with the clock displaying a Clock Price of 250 ECU. Then, after the market begins, for every half-second that passes, the Clock Price will decrease by 1 ECU. To express your willingness to purchase the unit at the current Clock Price, you must click the button labeled Purchase located beside the clock. The Clock Price at which you click Purchase will be referred to as your bid.

The clock will continue to tick down until the first moment that one of the participants in your market clicks Purchase. At this point, the clock will stop, and the participant who clicked Purchase will be the one to purchase the unit. The price displayed as the Clock Price at the time when that participant clicked Purchase will be the Market Price for that period.

In the event two or more participants in the same market click the Purchase button at the same price (i.e., a tie), one of the tying participants will be chosen at random to purchase the unit at that Market Price.

If no participant in a given market has clicked Purchase when the Clock Price reaches zero, then no participant in that market will purchase the unit of the commodity.
5. Those participants that do not purchase the item in a given period receive a Profit of zero for that trading period. The participant that purchases the item receives a Profit given by
Profit = Resale Value - Market Price.

Profits earned during a trading period will be added to the purchaser's Cash Balance, and losses incurred will be subtracted from the purchaser's Cash Balance. Remember, your goal should be to maximize your profits, not the number of times you outbid the other participants.

If your Cash Balance falls below zero, you will not be allowed to continue participating. However, you are permitted to bid in excess of your Cash Balance in any given period.
6. For each market, the Resale Value will be a randomly drawn number between 25 ECU and 225 ECU . Any value within this range has an equally likely chance of being drawn, and all randomizations are done all over again for each trading period.
7. Participants are NOT told the Resale Value in their market. However, at the beginning of each trading period, each participant will receive a Signal (which may be regarded as an estimate), that may provide information about the Resale Value in her/his market. Your signal will be randomly chosen between: a lower bound equal to the Resale Value minus 24, and an upper bound equal to the Resale Value plus 24. For example, if the Resale Value is 75 , then every participant in this market will receive a signal that is between 51 and 99. All signals in this interval are equally likely to be received, and a separate (and independent) randomization is done for each participant. That is, although all participants in the same market will face the same Resale Value, their signals that are generated in the way we have just described will usually be different.

Notice that you might receive a signal below 25 ECU or above 225 ECU. There is nothing strange about this; it just indicates that the Resale Value is either close to 25 ECU or close to 225 ECU.
8. Your signal will be displayed on your screen above the clock. This signal is strictly private information and is not to be revealed to anyone else. You are not to look at anyone else's screen or speak to any other participant while the experiment is in progress. This is important to the validity of the study.
9. As we mentioned earlier, Resale Values are determined randomly and independently from period to period. As such, a high value in one period tells you nothing about whether it might be high or low in the next period. Since a similar independent random process determines the signals, your signal in one period tells you nothing about signals or the Resale Value in later periods.
10. Information on the Computer Screen. At the beginning of each period, the top of the computer screen will show your ID number, your current Cash Balance, and the period number. In the middle of the screen, you will see your signal and the only possible Resale Values consistent with your signal. When everyone has clicked the Continue button, you will see a screen with the clock showing the initial Clock Price of 250 ECU. To the right of the clock will appear the Purchase button, and above the clock your signal for that period will be displayed. The auction will begin soon after seeing this screen, so you should plan your strategy when you first see your signal.

After each trading period is over, you will see a screen that summarizes the auction you just finished, showing: the Market Price, the Resale Value, the highest bidder's profit or loss, your profit or loss, your Cash Balance entering the period, and your current Cash Balance. Towards the bottom of the screen, you will see your signal, the signal of the participant who submitted the highest bid, and the signals of all buyers in your market. When all participants have clicked Continue, we proceed to the next period.
11. We will start with three or four practice "dry runs" that do not count towards your earnings, at which point we will stop and answer any additional questions.

Are there any questions?

## Auction Instructions for $S D$ treatment

1. We will now create a market in which you will act as buyers of a fictitious commodity in a sequence of trading periods. A single unit of the commodity will be auctioned off in each trading period. There will be several trading periods.
2. At the start of each period, participants are randomly placed into "markets" containing six (6) participants. If the number of available participants is not an exact multiple of six, there will be a small number of participants that are not placed in a market and who will sit out until the next period.

The random assignment of participants to markets will be done separately and independently for each period. This means that the chances that you and any given other participant are in the same market in the current period do not depend on whether the two of you were in the same market in previous periods. Rarely, this randomization procedure can have the same participant sit out for two or more consecutive periods.
3. Your task is to bid for the commodity in competition with the other five buyers in your market. In each period, the computer will select a RESALE VALUE for the unit being auctioned. This indicates the value to you of purchasing the unit in your market. This value may be thought of as the amount you would receive if you were to resell the unit. The process of determining resale values and your signals regarding resale values will be described in Sections 6 and 7 below.
4. In the middle of the screen, you will see a rectangular clock displaying a price in ECUs, which we refer to as the "Clock Price." In each new period, the market will begin with the clock displaying a Clock Price of 250 ECU. Then, after the market begins, for every half-second that passes, the Clock Price will decrease by 1 ECU. To express your willingness to purchase the unit at the current Clock Price, you must click the button labeled Purchase located beside the clock. The Clock Price at which you click Purchase will be referred to as your bid.

The clock will continue to tick down until you bid or the Clock Price reaches 0 . At that point, the results of the market will be displayed. The participant in each market who first clicked Purchase, and therefore clicked at the highest price, will be the one to purchase the unit. The price displayed as the Clock Price at the time when that participant clicked Purchase will be the Market Price for that period.

In the event two or more participants in the same market click the Purchase button at the same price (i.e., a tie), one of the tying participants will be chosen at random to purchase the unit at that Market Price.

If no participant in a given market has clicked Purchase when the Clock Price reaches zero, then no participant in that market will purchase the unit of the commodity.
5. Those participants that do not purchase the item in a given period receive a Profit of zero for that trading period. The participant that purchases the item receives a Profit given by
Profit = Resale Value - Market Price.

Profits earned during a trading period will be added to the purchaser's Cash Balance, and losses incurred will be subtracted from the purchaser's Cash Balance. Remember, your goal should be to maximize your profits, not the number of times you outbid the other participants.

If your Cash Balance falls below zero, you will not be allowed to continue participating. However, you are permitted to bid in excess of your Cash Balance in any given period.
6. For each market, the Resale Value will be a randomly drawn number between 25 ECU and 225 ECU . Any value within this range has an equally likely chance of being drawn, and all randomizations are done all over again for each trading period.
7. Participants are NOT told the Resale Value in their market. However, at the beginning of each trading period, each participant will receive a Signal (which may be regarded as an estimate), that may provide information about the Resale Value in her/his market. Your signal will be randomly chosen between: a lower bound equal to the Resale Value minus 24, and an upper bound equal to the Resale Value plus 24. For example, if the Resale Value is 75 , then every participant in this market will receive a signal that is between 51 and 99. All signals in this interval are equally likely to be received, and a separate (and independent) randomization is done for each participant. That is, although all participants in the same market will face the same Resale Value, their signals that are generated in the way we have just described will usually be different.

Notice that you might receive a signal below 25 ECU or above 225 ECU. There is nothing strange about this; it just indicates that the Resale Value is either close to 25 ECU or close to 225 ECU.
8. Your signal will be displayed on your screen above the clock. This signal is strictly private information and is not to be revealed to anyone else. You are not to look at anyone else's screen or speak to any other participant while the experiment is in progress. This is important to the validity of the study.
9. As we mentioned earlier, Resale Values are determined randomly and independently from period to period. As such, a high value in one period tells you nothing about whether it might be high or low in the next period. Since a similar independent random process determines the signals, your signal in one period tells you nothing about signals or the Resale Value in later periods.
10. Information on the Computer Screen. At the beginning of each period, the top of the computer screen will show your ID number, your current Cash Balance, and the period number. In the middle of the screen, you will see your signal and the only possible Resale Values consistent with your signal. When everyone has clicked the Continue button, you will see a screen with the clock showing the initial Clock Price of 250 ECU. To the right of the clock will appear the Purchase button, and above the clock your signal for that period will be displayed. The auction will begin soon after seeing this screen, so you should plan your strategy when you first see your signal.

After each trading period is over, you will see a screen that summarizes the auction you just finished, showing: the Market Price, the Resale Value, the highest bidder's profit or loss, your profit or loss, your Cash Balance entering the period, and your current Cash Balance. Towards the bottom of the screen, you will see your signal, the signal of the participant who submitted the highest bid, and the signals of all buyers in your market. When all participants have clicked Continue, we proceed to the next period.
11. We will start with three or four practice "dry runs" that do not count towards your earnings, at which point we will stop and answer any additional questions.

Are there any questions?

## Auction Instructions for $F P$ Treatment

1. We will now create a market in which you will act as buyers of a fictitious commodity in a sequence of trading periods. A single unit of the commodity will be auctioned off in each trading period. There will be several trading periods.
2. At the start of each period, participants are randomly placed into "markets" containing six (6) participants. If the number of available participants is not an exact multiple of six, there will be a small number of participants that are not placed in a market and who will sit out until the next period.

The random assignment of participants to markets will be done separately and independently for each period. This means that the chances that you and any given other participant are in the same market in the current period do not depend on whether the two of you were in the same market in previous periods. Rarely, this randomization procedure can have the same participant sit out for two or more consecutive periods.
3. Your task is to bid for the commodity in competition with the other five buyers in your market. In each period, the computer will select a RESALE VALUE for the unit being auctioned. This indicates the value to you of purchasing the unit in your market. This value may be thought of as the amount you would receive if you were to resell the unit. The process of determining resale values and your signals regarding resale values will be described in Sections 6 and 7 below.
4. In each period, in the middle of the computer screen, you will see two rectangular boxes. You will be asked to type a number between 0 and 250 into the first box, which will be referred to as your bid for that period. Your bid should be an integer. To avoid possible typing mistakes, you will also be asked to confirm your bid by typing the same number into the second box. When you have entered your bid in both boxes, please click on the Continue button.

After everyone in your market has submitted their bids, the results of the market will be displayed. The participant in each market who submits the highest bid will be the one to purchase the unit, and the highest bid will be the Market Price for that period.

In the event two or more participants in the same market each submit the highest bid (i.e., a tie), one of the tying participants will be chosen at random to purchase
the unit at that Market Price.
5. Those participants that do not purchase the item in a given period receive a Profit of zero for that trading period. The participant that purchases the item receives a Profit given by

$$
\text { Profit }=\text { Resale Value }- \text { Market Price. }
$$

Profits earned during a trading period will be added to the purchaser's Cash Balance, and losses incurred will be subtracted from the purchaser's Cash Balance. Remember, your goal should be to maximize your profits, not the number of times you outbid the other participants.

If your Cash Balance falls below zero, you will not be allowed to continue participating. However, you are permitted to bid in excess of your Cash Balance in any given period.
6. For each market, the Resale Value will be a randomly drawn number between 25 ECU and 225 ECU . Any value within this range has an equally likely chance of being drawn, and all randomizations are done all over again for each trading period.
7. Participants are NOT told the Resale Value in their market. However, at the beginning of each trading period, each participant will receive a Signal (which may be regarded as an estimate), that may provide information about the Resale Value in her/his market. Your signal will be randomly chosen between: a lower bound equal to the Resale Value minus 24, and an upper bound equal to the Resale Value plus 24. For example, if the Resale Value is 75 , then every participant in this market will receive a signal that is between 51 and 99. All signals in this interval are equally likely to be received, and a separate (and independent) randomization is done for each participant. That is, although all participants in the same market will face the same Resale Value, their signals that are generated in the way we have just described will usually be different.

Notice that you might receive a signal below 25 ECU or above 225 ECU. There is nothing strange about this; it just indicates that the Resale Value is either close to 25 ECU or close to 225 ECU.
8. Your signal will be displayed on your screen above the Bid Box and the Confirmation Box. This signal is strictly private information and is not to be revealed to anyone else. You are not to look at anyone else's screen or speak to any other
participant while the experiment is in progress. This is important to the validity of the study.
9. As we mentioned earlier, Resale Values are determined randomly and independently from period to period. As such, a high value in one period tells you nothing about whether it might be high or low in the next period. Since a similar independent random process determines the signals, your signal in one period tells you nothing about signals or the Resale Value in later periods.
10. Information on the Computer Screen. At the beginning of each period, the top of the computer screen will show your ID number, your current Cash Balance, and the period number. In the middle of the screen, you will see your signal and the only possible Resale Values consistent with your signal. When everyone has clicked the Continue button, you will see a screen with the Bid Box and the Confirmation Box. To the right of these boxes will appear the Continue button, and above these boxes your signal for that period will be displayed.

After each trading period is over, you will see a screen that summarizes the auction you just finished, showing: the Market Price, the Resale Value, the highest bidder's profit or loss, your profit or loss, your Cash Balance entering the period, and your current Cash Balance. Towards the bottom of the screen, you will see your signal, the signal of the participant who submitted the highest bid, and the signals of all buyers in your market. When all participants have clicked Continue, we proceed to the next period.
11. We will start with three or four practice "dry runs" that do not count towards your earnings, at which point we will stop and answer any additional questions.

Are there any questions?

## Auction Instructions for $A D^{P V}$ Treatment

(instructions for $S D^{P V}$ and $F P^{P V}$ treatments available upon request)

1. We will now create a market in which you will act as buyers of a fictitious commodity in a sequence of trading periods. A single unit of the commodity will be auctioned off in each trading period. There will be several trading periods.
2. At the start of each period, participants are randomly placed into markets containing six (6) participants. If the number of available participants is not an exact multiple of six, there will be a small number of participants that are not placed in a market and who will sit out until the next period. The random assignment of participants to markets will be done separately and independently for each period. This means that the chances that you and any given other participant are in the same market in the current period do not depend on whether the two of you were in the same market in previous periods. Rarely, this randomization procedure can have the same participant sit out for two or more consecutive periods.
3. Your task is to bid for the commodity in competition with the other five buyers in your market. In each period, the computer will select your PRIVATE VALUE for the unit being auctioned. This indicates the value to you of purchasing the unit in your market. This value may be thought of as the amount you would receive if you were to resell the unit back to us. The process of determining private values will be described in Sections 6 and 7 below.
4. In each period, in the middle of the computer screen, you will see a rectangular clock displaying a price in ECUs, which we refer to as the Clock Price. In each new period, the market will begin with the clock displaying a Clock Price of 250 ECU. Then, after the market begins, for every half-second that passes, the Clock Price will decrease by 1 ECU. To express your willingness to purchase the unit at the current Clock Price, you must click the button labeled Purchase located beside the clock. The Clock Price at which you click Purchase will be referred to as your bid.

The clock will continue to tick down until the first moment that one of the participants in your market clicks Purchase. At this point, the clock will stop, and the participant who clicked Purchase will be the one to purchase the unit. The price displayed as the Clock Price at the time when that participant clicked Purchase will be the Market Price for that period.

In the event two or more participants in the same market click the Purchase button at the same price (i.e., a tie), one of the tying participants will be chosen at random to purchase the unit at that Market Price.

If no participant in a given market has clicked Purchase when the Clock Price reaches zero, then no participant in that market will purchase the unit of the commodity.
5. Those participants that do not purchase the item in a given period receive a Profit of zero for that trading period. The participant that purchases the item receives a Profit given by

$$
\text { Profit }=\text { Private Value }- \text { Market Price. }
$$

Profits earned during a trading period will be added to the purchaser's Cash Balance, and losses incurred will be subtracted from the purchaser's Cash Balance. Remember, your goal should be to maximize your profits, not the number of times you outbid the other participants.

If your Cash Balance falls below zero, you will not be allowed to continue participating. However, you are permitted to bid in excess of your Cash Balance in any given period.
6. For each market, we will first draw a number, which will be the SEED that helps us determine the Private Values of the participants in that market (to be explained below). The Seed will be a randomly drawn number between 25 ECU and 225 ECU. Any value within this range has an equally likely chance of being drawn, and all randomizations are done all over again for each trading period.
7. Participants are not told the Seed in their market. However, your Private Value will be randomly chosen between: a lower bound equal to the Seed minus 24, and an upper bound equal to the Seed plus 24. For example, if the Seed drawn happens to be 75, then the Private Values of each participant in this market will be between 51 and 99. All Private Values in this interval are equally likely to be received, and a separate (and independent) randomization is done for each participant. That is, although all participants in the same market will have the same Seed, their Private Values that are generated in the way we have just described will be different. Notice that you might receive a Private Value below 25 ECU or above 225 ECU. There is
nothing strange about this; it can happen if the Seed is either close to 25 ECU or close to 225 ECU.
8. Your Private Value will be displayed on your screen above the clock. This Private Value is strictly private information and is not to be revealed to anyone else. You are not to look at anyone else's screen or speak to any other participant while the experiment is in progress. This is important to the validity of the study.
9. As we mentioned earlier, Seeds are determined randomly and independently from period to period. As such, a high value in one period tells you nothing about whether it might be high or low in the next period. Since a similar independent random process determines the Private Values, your Private Value in one period tells you nothing about the Private Values in later periods.
10. Information on the Computer Screen. At the beginning of each period, the top of the computer screen will show your ID number, your current Cash Balance, and the period number. In the middle of the screen, you will see your Private Value. When everyone has clicked the Continue button, you will see a screen with the clock showing the initial Clock Price of 250 ECU. To the right of the clock will appear the Purchase button, and above the clock your Private Value for that period will be displayed. The auction will begin soon after seeing this screen, so you should plan your strategy when you first see your Private Value. After each trading period is over, you will see a screen that summarizes the auction you just finished, showing: the Market Price, the Private Value of the highest bidder, the highest bidder's profit or loss, your Cash Balance entering the period, and your current Cash Balance. Towards the bottom of the screen, you will see your Private Value, the Private Value of the participant who submitted the highest bid, the Private Values of all participants in your market, and the Seed in your market. When all participants have clicked Continue, we proceed to the next period.
11. We will start with three or four practice "dry runs" that do not count towards your earnings, at which point we will stop and answer any additional questions.

Are there any questions?

Screen Shot from Card Tasks, Question 2


Screen Shot from $A D$ Auction


Screen Shot from FP Auction


Screen Shot from $S D^{P V}$ Auction


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School of Economics and Finance
Queen Mary University of London
Mile End Road
London E1 4NS
Tel: +44 (0)20 78827356
Fax: +44 (0)20 89833580
Web: www.econ.qmul.ac.uk/research/workingpapers/


[^0]:    *Dan Levin: Department of Economics, The Ohio State University, 1945 N. High Street, 410 Arps Hall, Columbus, OH 43210-1172, USA; levin.36@osu.edu. James Peck: Department of Economics, The Ohio State University, 1945 N. High Street, 410 Arps Hall, Columbus, OH 43210-1172, USA; peck.33@osu.edu. Asen Ivanov: School of Economics and Finance, Queen Mary, University of London, 327 Mile End Road, London E1 4NS, United Kingdom; a.ivanov@qmul.ac.uk. This work is supported by the NSF under Grants No. SES-1031101 (for Levin and Peck) and No. SES-1030467 (for Ivanov). Any opinions, findings and conclusions or recommendations expressed are those of the authors and do not necessarily reflect the views of the NSF. Asen Ivanov's primary role was to run some of the sessions and help with the data analysis. We thank Alex Gotthard Real for programming, help running the sessions, and valuable discussions. We thank Steve Cosslett and David Harless for helpful conversations, and Siqi Pan for help running the private value sessions. We also thank two referees for suggesting the private-value treatments and other helpful suggestions.

[^1]:    ${ }^{1}$ Earlier versions of this paper used the term "insight" or "non-computational reasoning." We now use the term "non-probabilistic reasoning" (NPR) in order to more accurately describe the distinct skills needed in our auctions and our card tasks (see below). BU may or may not involve computations, so the distinction is about reasoning about probabilities vs. reasoning about something unrelated to probabilities.
    ${ }^{2}$ Indeed, a subject without NPR skills may not even consider any expected value benchmark, and

[^2]:    ${ }^{5}$ For cursed equilibrium, see Eyster and Rabin (2005). For level-k beliefs, see Nagel (1995), Stahl and Wilson (1994, 1995), Camerer, Ho, and Chong (2004), and Crawford and Iriberri (2007). For analogy-based expectations, see Jehiel (2005, 2008).
    ${ }^{6}$ See McKelvey and Palfrey $(1995,1998)$.

[^3]:    ${ }^{7}$ We are not sure how to explain why our private value results differ from Turocy, Watson, and Battalio (2007), but the following distinction may be important. In our affiliated private values design, most private values give the subject very little information about how her value compares to the values of the other subjects. On the other hand, with independent private values, a subject whose value is in the bottom half of the distribution knows that she is very unlikely to be in contention to win the auction in the SD and FP treatments, but not in the AD treatment once the clock price drops below her value.

[^4]:    ${ }^{8}$ Regarding the BNE bidding function in the range $x<\underline{x}+\varepsilon$, see footnote 8 in Kagel and Levin (1986). A solution for $x>\bar{x}-\varepsilon$ was derived in Kagel and Richard (2001).

[^5]:    ${ }^{9}$ Our $S D$ serves as a control for a pure format effect due to the dynamic structure. It is also possible that a psychological affect such as "excitement" or "suspense" is operating in the $A D$ but not in the $F P$ or $S D$. However, we find no treatment differences in our private-value auctions, suggesting that excitement or suspense is not a factor.

[^6]:    ${ }^{10}$ Subjects were asked to answer in percentage terms, so a $2 / 3$ probability would be answered as 66 or 67.
    ${ }^{11}$ One can arrive at the correct answer of 100 percent by pure logic, without computations, which is arguably a different sort of probabilistic reasoning than if black were the face up color. For this reason, we will have 2 dummy variables for BU skills, depending on which color was face up on Q1.
    ${ }^{12}$ The same reasoning would lead to a willingness to pay of zero, even if the DM exhibits a failure of BU. Suppose the DM mistakenly believes that when the face-up side is black, then the face-down side is black with probability one-half. Then the DM is indifferent between guessing black and white when the face-up side is black, and strictly prefers to guess black when the face-up side is white. (If the DM is "anti-Bayesian" and assigns a probability less than one half when observing black face up, then the willingness to pay would be positive. Few subjects do this on Q1.)

[^7]:    ${ }^{13}$ An advantage of the three-card deck for measuring BU is that the DM does not reach the correct answer by "counting colors." Suppose the face-up side is black. If one reasons that there are two black sides and three white sides that I do not see, this yields the incorrect probability of $2 / 5$ and anti-Bayesian updating in the wrong direction. For the two-card deck, this logic actually yields the correct probability of $2 / 3$.

[^8]:    ${ }^{14}$ One session (not listed) experienced a hardware failure and was aborted. Also, subjects 358-364 did not perform the card tasks due to a computer glitch.
    ${ }^{15}$ Subjects could go through each question at their own pace. Once a subject answered a question, she could not go back to change her answer. Subjects went through the questions at very different speeds, but took no more than 5-6 minutes per question.
    ${ }^{16}$ If the number of subjects was not a multiple of six, some randomly selected subjects sat out during that period, so all markets had exactly six bidders.
    ${ }^{17}$ The number of dry run periods was chosen to give most subjects a chance to participate in two dry run auctions, and each subject a chance to participate in at least one dry run auction. If the number of subjects was a multiple of 6 , no one sat out and we ran 2 dry runs. If a subject was randomly selected to sit out during the first three dry runs, we ran a fourth dry run.
    ${ }^{18}$ Due to time constraints, session 12 had to be stopped after period 27.

[^9]:    ${ }^{19}$ The only exception is session 1 (common values), where the initial balance was 15 ECU. Lower initial cash balances were provided in the private-value treatments because a subject could avoid bankruptcy simply by never bidding above her private value.
    ${ }^{20}$ In the $A D / S D / F P, 26 / 40 / 38$ percent of subjects were bankrupt by the end of the session. In the $A D^{P V} / S D^{P V} / F P^{P V}, 13 / 36 / 17$ percent of subjects were bankrupt by the end of the session.
    ${ }^{21}$ Earnings equal the maximum of the bankruptcy payment and the cash balance at the end of the experiment: initial balance + card task earnings $+/-$ auction profits/losses. Average earnings on the card tasks alone in the $A D / S D / F P / A D^{P V} / S D^{P V} / F P^{P V}$ treatment are $\$ 2.83 / \$ 3.03 / \$ 2.98 / \$ 2.76 / \$ 2.55 / \$ 3.04$.

[^10]:    ${ }^{22}$ In the $A D / S D / F P / A D^{P V} / S D^{P V} / F P^{P V}$ auctions, 3.7/4.2/2.9/2.2/6.3/2.9 percent of highest bids are strictly above $x+\varepsilon$. In the $S D / F P / S D^{P V} / F P^{P V}$ auctions, there are $0 / 1 / 1 / 0$ non-highest bids strictly above $x+\varepsilon$.
    ${ }^{23}$ Highest bids are never below $x-2 \varepsilon$. In the $S D / F P / S D^{P V} / F P^{P V}$ auctions, 1.5/4.2/0.5/0.6 percent of bids are strictly below $x-2 \varepsilon$. In the analysis below, keeping bids strictly below $x-2 \varepsilon$ will not change the main regression results of Table 4 or Table 5.

[^11]:    ${ }^{24}$ Our finding that $60 \%$ of subjects give a probability of 0.5 in Q1 with black face-up and in Q3 is very similar to what Page (1998) finds in the isomorphic "let's make a deal" problem. See also

[^12]:    Friedman (1998).
    ${ }^{25}$ For example, a subject observing black face-up might reason incorrectly that there are 3 white sides and 2 black sides that she is not observing, so that white is more likely than black.

[^13]:    ${ }^{26}$ We do not use highest bids which do not win the object (as a result of tying and losing the coin toss) because such bids lead to a profit of 0 purely due to luck (or bad luck).

[^14]:    ${ }^{27}$ For example, suppose the error term has the random effects structure $u_{i t}=v_{i}+e_{i t}$. Then, a subject with a low $v_{i}$ will also tend to have low cash balances (because she will be bidding high, winning, and making losses). Note that we cannot use fixed effects estimation because the variables of main interest (the auction format dummies) are not time-varying.

[^15]:    ${ }^{28}$ Note that our regressions include only observations coming from bidders who are not bankrupt. Formally, our instrumental variables regression does not take into account this selection.
    ${ }^{29}$ For subjects who saw white/black face-up on $\mathrm{Q} 1, Q 1 b Q 3 / Q 1 w Q 3$ is set equal to 0 . Also, given that, as explained below, we will not be using data from the $A D$ treatment in this regression, the variables $S D$ and $t \times S D$ that were present in Regression (3) are now dropped due to collinearity.

[^16]:    ${ }^{30}$ For example, suppose there is an actual positive effect of, say, $Q 1 b Q 3$ on shading rates. Then when the highest bidder is someone with $Q 1 b Q 3=1$, she will on average have a lower error $u_{i t}$ than when the highest bidder is someone with $Q 1 b Q 3=0$. But then $Q 1 b Q 3$ and the error are correlated when we look only at highest bidders, so that our coefficient estimates would be inconsistent.
    ${ }^{31}$ In computing this variable, we do not drop auctions in which the subject's bid is above $x+\varepsilon$ or below $x-2 \varepsilon$ if the subject was not the winning bidder. If the subject does not win, she earns zero profit and there is no possibility for very high or very low bids to distort MeanProfit.
    ${ }^{32}$ In the PV sessions, there is no evidence for changing shading rates or profits across periods in any of the auction formats. See the coefficients on $t, t \times S D$, and $t \times F P$ in Table 5 .

[^17]:    ${ }^{33}$ The coefficient on $t$ in the first and second regression equals $0.007(p<0.01)$ and 0.006 ( $p=$ 0.022 ), respectively.

