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Horizontal and Vertical Polarization: Task-Specific Technological Change in a Multi-Sector Economy*

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Abstract

We construct a multi-layer model of skills, occupations, and sectors. Technological progress among middle-skill occupations raises the employment shares and relative wages of lower- and higher-skill occupations (*horizontal* polarization), and those of managers over workers (*vertical* polarization). Polarization is faster within sectors that rely more on middle-skill workers, endogenously boosting their TFP. This shrinks their employment and output shares (structural change) if sector outputs are complementary. We empirically validate our theoretical predictions, and show that task-specific technological progress—which was faster for routine-manual tasks and slower for interpersonal tasks—played a major role in transforming the U.S. economy since 1980.

JEL codes: J24, J31, L16, O14, O33

Keywords: job polarization, structural change, wage inequality

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Two of the most conspicuous changes in many advanced economies since 1980 are polarization (shrinking middle-class jobs relative to lower- and higher-paying jobs) and deindustrialization (shrinking manufacturing relative to services). How are they related? Many policymakers implicitly assume that the two are the same, or that polarization results from deindustrialization, thus promising to bring back middle-class jobs by subsidizing and protecting the manufacturing sector. On the other hand, one may think that the two are only superficially related, since middle-class jobs began to lose ground in the 1980s while deindustrialization began as far back as the 1950s.

In this paper, we develop a novel framework that integrates the occupational and industrial structures of an economy with its distribution of individual skills, which provides a comprehensive view on the forces shaping them. We apply this framework to the debate on polarization and deindustrialization.

In our model, individuals are heterogeneous in two dimensions of skill—managerial talent and worker human capital—based on which they become a manager or a worker. Workers then select into a continuum of tasks (or occupations) based on their human capital.[1] Managers organize the workers' tasks, and combine it with their own task output to produce sector-specific goods. Sectors differ in how intensively they use different tasks in production. Skills are sector-neutral, so individuals care only about their occupation and are indifferent about which sector they work in. Despite these many layers, the model remains tractable enough for us to derive analytical results.

Our main theoretical contribution is to characterize the equilibrium assignment across occupations and sectors, and to prove a series of comparative statics in response to technological progress among a set of tasks (task-specific technological change, or TSTC hereafter) that is sector- and factor-neutral. We show that if different tasks are complementary in production, faster technological progress among middle-skill tasks—more precisely, tasks chosen by middle-skill workers in an initial equilibrium, which correspond to middle-class jobs in 1980—leads to: (i) job and wage polarization among workers; (ii) a rise in managers' employment share and mean wage relative to workers as a whole, which we dub *vertical* polarization to distinguish from the *horizontal* polarization across workers; (iii) faster horizontal and vertical polarization within sectors that depend more on middle-skill tasks and less on managers; and (iv) endogenously faster total factor productivity (TFP) growth of such sectors, shrinking their employment and value-added shares if sectoral goods are complementary (i.e., structural change).

[1] Technically, a (set of) task(s) is the technology used by a certain occupation. Nonetheless, we will use "task" and "occupation" interchangeably throughout the paper.

In addition to (i), which is well-documented in the literature (Autor and Dorn, 2013), we empirically establish that predictions (ii) to (iv) are salient features of the U.S. economy since 1980. Specifically, we document that manufacturing is more reliant on middle-skill workers and less on managers than services, that both sectors have polarized horizontally and vertically, and that both dimensions of polarization are faster within manufacturing than in services. It is also well understood that the faster growth of manufacturing's TFP—which accelerated around 1980 in the data—is an important driver of structural change from manufacturing to services.

The last result merits further discussion. First, sector-level TFP in our model is endogenously determined by the within-sector allocation of skills across occupations. So as long as different sectors use some tasks more intensively than others, technological progress among certain tasks—even though sector neutral—has differential impact across sectors, causing structural change. Second, if TSTC is the only source of structural change, those jobs with faster progress may vanish asymptotically, but all sectors must coexist: Once the employment shares of the jobs with faster progress become negligible, structural change ceases. This contrasts with theories of structural change that rely on sector-specific forces, in which the shift of production factors from one sector to another continues until the shrinking sector vanishes.

The explicit modeling of managers as a special occupation allows us to analyze the organizational structure of an economy—how managers and workers form establishments for production—in conjunction with its occupational and sectoral allocations. In the data, manufacturing establishments grew faster in value-added but shrank by more in employment than service establishments. This is what our model predicts in response to TSTC among middle-skill worker occupations. Since manufacturing uses middle-skill jobs more intensively, its establishment productivity and value-added rise faster than service establishments'. For the same reason, vertical polarization (i.e., fewer workers per manager) is faster within manufacturing than in services, resulting in a relatively larger drop in employment per establishment for manufacturing, assuming that the number of managers per establishment remains stable over time.[2]

The theoretical model has one managerial task and a continuum of worker tasks. To quantify how much TSTC can account for the data, we discretize the latter into 10 occupation categories. We find that, from 1980 to 2010, TSTC is most pronounced for middle-skill jobs, reducing their employment shares and relative wages. At the same time, because these jobs are a relatively larger share of employment in manufacturing

[2] In our model, a single manager hires multiple workers to form a team for production. Of course in reality, an establishment may have multiple managers and teams.

than in services, manufacturing's TFP endogenously grows relative to services. Surprisingly, the magnitude of manufacturing's relative TFP growth that is endogenously generated by TSTC almost exactly shrinks its employment share as is observed in the data. Thus TSTC alone—without any sector- or factor-specific technological change—almost fully accounts for the changes in the occupational, sectoral, and organizational structure of the U.S. economy since 1980.

The main message of our quantitative analysis is not that polarization and structural change from manufacturing to services are hardwired to coincide, but that differential productivity growth across tasks can manifest as large employment shifts across sectors. As already mentioned, when the jobs with the fastest TSTC are reduced to negligible employment shares, manufacturing's employment share ceases to decline. In fact, it may even recover if jobs with the second highest rate of TSTC are used more intensively in services. That is, in our model, a reversal in structural change can occur even when TSTC and polarization continue in the same direction. This is indeed consistent with the recent uptick in manufacturing employment in the U.S. (often attributed to "reshoring").

Our finding that underscores the role of TSTC has an important, novel policy implication. Even if pro-manufacturing policies (e.g., subsidies and protectionism) were to revive manufacturing jobs, they would not bring back those middle-class jobs of the past made obsolete by TSTC. To the contrary, since polarization was more severe within the manufacturing sector, such policies may even further destroy middle-class jobs in the overall economy.

Of course, the direction of TSTC may shift over the longer term. For example, in earlier periods, technological progress may have been faster for tasks or jobs paying less than middle-class jobs (such as farming or low-paying manufacturing jobs), in which case the sectors that rely more intensively on those jobs would have shrunk (e.g., first agriculture and then manufacturing). Such structural change would have been accompanied by the expansion of middle- and high-paying jobs at the expense of low-paying jobs, not by polarization.

The next natural question to ask is what explains such differential productivity improvements across tasks since 1980. Autor and Dorn (2013), Goos, Manning, and Salomons (2014) and others have hypothesized that "routinization," or faster technological advancement for tasks that are more routine in nature (which tend to be middle-skill occupations in the data), led to (horizontal) polarization. They test this empirically by constructing a routine-task intensity (RTI) index for each occupation from the Dictionary of Occupation Titles (DOT) and its successor O*NET.

We follow a similar route, but use more disaggregated indices than RTI that consider detailed characteristics of occupations. We find that the degree of TSTC we quantify from the changes in each of our 11 occupation groups is much more strongly correlated with the routine-manual index (a component of RTI) and with the inverse of the manual-interpersonal index than with RTI.

In other words, technological progress since 1980 is primarily embodied in manual tasks that are repetitive in nature and require few interpersonal skills. This strongly suggests that the relatively easier automation of such tasks explains their faster productivity growth, which is consistent with the routinization hypothesis. On the other hand, the fact that polarization is observed even within the service sector suggests that trade may not have been the main driver.

Related literature The model we consider is of particular relevance for the U.S. and other advanced economies. The 1980s marks a starting point of rising labor market inequality, of which polarization is a salient feature. It also coincided with the rise of low-skill service jobs (Autor and Dorn, 2013) and a clear rise in manufacturing productivity. Our main finding in this regard is that task-specific technological progress is of first-order importance for understanding the observed changes not only across occupations but also in the industrial structure of the economy.

Costinot and Vogel (2010) presents a task-based model in which workers with a continuum of one-dimensional skill sort into a continuum of tasks. The worker side of our model is similar to theirs (except that we include capital), but we gain new insights by incorporating two dimensional skills (managerial talent and worker human capital) and multiple sectors.

The only other paper we know of with a structure in which individuals with different skills sort into occupations, which are then used as production inputs in multiple sectors, is Stokey (2016). The within-sector side of its model can be described as a version of Costinot and Vogel (2010), in which skills are continuous but tasks are discrete. The latter assumption enables an analytic characterization of the effect of TSTC, which is in turn used for demonstrating the broad range of phenomena that can be explained by such a model. We take the same approach in our quantitative section (i.e., tasks are discretized), but emphasize the differential pace of polarization across sectors and explicitly relate polarization to structural change. In particular, we use U.S. data to quantify how relevant our model is for employment and relative wage trends across occupations and sectors between 1980 and 2010. In addition, we treat managers as an occupation that is qualitatively different from workers, so the model has implications for how production is organized in different sectors. All our modeling assumptions are backed by several new facts that we document.

The manager-level technology in our model extends the span-of-control model of Lucas (1978), in which managers hire workers for production. Unlike all existing variants of the span-of-control model, our managers organize tasks instead of workers. That is, instead of deciding how many workers to hire, they decide on the quantities of each task to use in production, and for each task, how much skill to hire. Moreover, we assume a constant-elasticity-of-substitution (CES) technology between managerial and worker tasks.[3]

Goos et al. (2014) empirically identifies relative price changes in task-specific capital as the main driver of employment polarization in Europe. It decomposes employment polarization into within- and between-industry components, but abstracts from changes in equilibrium wages and aggregate quantities. Our analysis shows that general equilibrium considerations have important implications for the estimation of the elasticity of substitution across tasks, a key parameter in such analyses.

Dürnecker and Herrendorf (2017) also considers occupations and industries, and show that structural change from manufacturing to services can be explained by shifts at the occupation level in many countries. Its conclusion is based on classifying occupations in the data as (mutually-exclusive) manufacturing or service jobs. In contrast, we characterize the selection of skills into the whole spectrum of occupations, and analyze the distinct effect of task-specific and sector-specific technological change for employment and wage inequalities over the entire skill distribution.

Structural change in our model occurs because sectoral productivities evolve differentially over time, as in Ngai and Pissarides (2007) and most other production-driven models of structural change. What we add to this literature is a mechanism for sectoral productivities to evolve *endogenously*, through changes in equilibrium occupational choices stemming from TSTC. Also related is Acemoglu and Guerrieri (2008), in which the capital-intensive sector vanishes in the limiting balanced growth path. In comparison, sectors in our model differ in how intensively they use different tasks. By contrasting different types of labor, rather than capital and labor, we can connect structural change across sectors to labor market inequality across occupations and skills. Moreover, ours implies that it is certain occupations rather than broadly-defined sectors that may vanish in the limit.

Finally, we note that some recent papers consider the relationship between skill and

[3]While Lucas's original model is based on a generic homogeneous-of-degree-one technology, virtually all papers that followed assume a Cobb-Douglas technology. In contrast, we incorporate (i) non-unitary elasticity between managers and workers, (ii) heterogeneity in worker productivity as well as in managerial productivity, (iii) multiple worker tasks (or occupations), and (iv) multiple sectors.

structural change. Buera and Kaboski (2012) and Buera, Kaboski, and Rogerson (2015) feature multiple worker types as different inputs of production. Similarly, Bárány and Siegel (2018) argues that polarization may be explained by structural change, in a model where skills are occupation-specific and occupations are sector-specific. In these models, task-specific technology is ruled out, so all change must be either skill- or sector-specific. The addition of the task dimension in our model separates skills from the occupation in which they are used, thereby permitting technological changes specific to a task and also an analysis of their impact on the selection of skills into occupations. It also allows us to exploit data on occupational employment and wages *within* sectors as well as across sectors.[4] Equally important, the sectoral TFPs in our model are endogenously determined by equilibrium occupational choices.

The rest of the paper is organized as follows. In Section 1, we summarize the relevant empirical facts. In Section 2, we present the model and solve for its equilibrium. In Section 3, we perform comparative statics showing that faster technological progress for middle-skill tasks leads to horizontal and vertical polarization, and to structural change. In Sections 4 and 5, we quantify the importance of TSTC and maps it into empirical measures of task characteristics. Section 6 concludes, outlining the broader applicability of our novel framework.

1 Facts

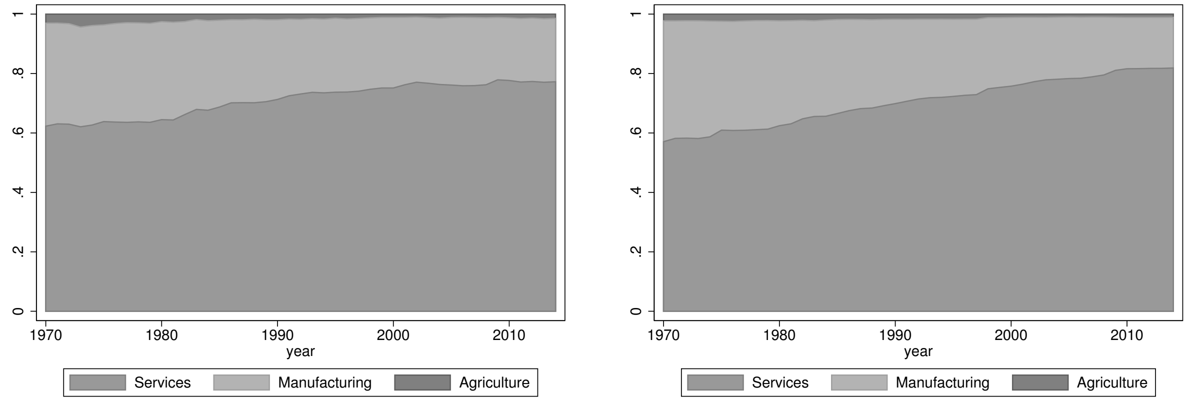
Here we summarize known facts on structural change and polarization, and present new evidence on how the two may be related. We consider managers as qualitatively different from all other occupations, and later relate them to establishments.

Structural change Figure 1 shows the (real) value-added output and employment share trends of three broadly defined sectors: agriculture, manufacturing and services, from 1970 to 2013. Following convention, e.g. Herrendorf, Rogerson, and Valentinyi (2014), "manufacturing" is the aggregation of manufacturing, mining and construction, and "services" the sum of all service and government sectors. The data are from the National Accounts published by the Bureau of Economic Analysis (BEA). In particular, employment is based on National Income and Product Accounts (NIPA) Table 6 (persons involved in production), counted in terms of full-time equivalent workers.[5]

Two facts are well documented in the literature. First, starting from even before

[4] Autor, Katz, and Kearney (2006), Acemoglu and Autor (2011) and Lee, Shin, and Lee (2015) show that residual wage inequality controlling for education groups is much larger and has risen more since 1980 than between-group inequality.

[5] Computing employment shares from the decennial census yields more or less the same trend.



(a) GDP, Value-Added

(b) Aggregate Employment Shares

Fig. 1: Structural Change, 1970-2013.
Source: BEA NIPA accounts. "Manufacturing" combines manufacturing, mining and construction, and services subsumes service and government. Sectoral output is computed via cyclical expansion from the industry accounts as in Herrendorf, Rogerson, and Valentinyi (2013). Employment is based on full-time equivalent number of persons in production in NIPA Table 6. Further details are in Section 4.2.

1970, agriculture was a negligible share of the U.S. economy. For the remainder of this paper, we drop agriculture, and all moments are computed as if the economy consisted only of manufacturing and services.

Second, structural change—the shifting of GDP and employment from manufacturing to services—exhibits a smooth trend starting from at least the 1970s, as noted in Herrendorf et al. (2014). Moreover, either sector's GDP share and employment share are almost identical both in terms of levels and trends. This implies a nearly constant input share of labor across the two sectors, as is assumed in our theoretical model.

In Figure 2, we show TFP of manufacturing and services from 1948 assuming a capital income share of 0.361.[6] Note that manufacturing's TFP relative to services was more or less constant prior to the early 1980s, after which it increased fast.[7] In our quantitative analysis, we relate this to faster task-specific technological progress among middle-skill jobs, which began to show a distinct declining trend around the same time (Autor and Dorn, 2013).

Job and wage polarization The rest of our empirical analysis is primarily based on the decennial U.S. Censuses 1980–2010, for which we closely follow Autor and

[6]This is the longest time period allowed by the industry accounts, which we need to compute real GDP and capital at the sector level. Herrendorf, Herrington, and Valentinyi (2015) argues that Cobb-Douglas sectoral production functions with equal capital income shares can quantitatively capture the effect of differentially evolving sectoral TFP's. See Section 4.2 for details.

[7]Herrendorf et al. (2014) also notes that manufacturing's relative TFP has grown quicker than services post 1970s, but that such trends may not be stable over a longer horizon.

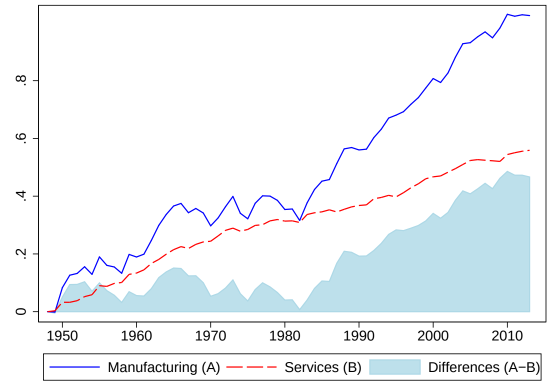


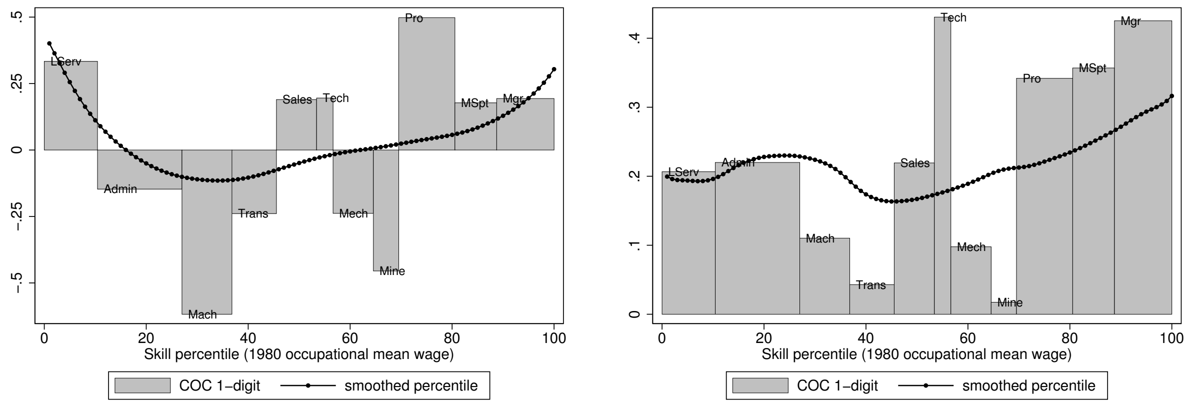
Fig. 2: Log Sectoral TFP, 1947–2013.
Source: BEA NIPA accounts. Sectoral output and capital are computed via cyclical expansion from the industry accounts as in Herrendorf et al. (2013). Employment is based on full-time equivalent number of persons in production in NIPA Table 6. Within each sector, TFP is measured as the Solow residual given a capital income share of 0.361, and log-TFP's are normalized to 0 in 1947.

Dorn (2013). We restrict our sample to 16–65 year-old non-farm employment. Figure 3 plots employment and wage changes by occupation from 1980 to 2010, extending Figure 1 in Autor and Dorn (2013) who considered changes up to 2005. Occupations are sorted into employment share percentiles by skill along the x-axis, where skill is proxied by the mean (log) hourly wage of each occupation in 1980. We follow the three-digit `occ1990dd` occupation coding convention in Dorn (2009), which harmonizes the `occ1990` convention in Meyer and Osborne (2005). This results in 322 occupation categories for which employment is positive in all 4 censuses. Employment is defined as the product of weeks worked times usual weekly hours.

The data is presented in two ways. First, following Autor and Dorn (2013), each dot in Figure 3 represents one percent of employment in 1980. The y-axis in Panel (a) measures each skill percentile's employment change from 1980 to 2010 in percentage points, and in Panel (b) the change in its mean wage. The changes are smoothed into percentiles across neighboring occupations using a locally weighted smoothing regression. Despite the Great Recession happening between 2005 and 2010, the long-run patterns are virtually the same as in their study: employment has shifted from the middle toward both lower- and higher-skill jobs. Likewise, wages have risen the least in the middle, and much more at the top.

Second, we group all occupations into 11 broad categories, mostly corresponding to the one-digit Census Occupation Codes (COC). These groups are ordered by the mean wage of each broadly defined group. To represent the groups in skill percentiles, the width of each group along the x-axis is set equal to its 1980 employment share.[8]

[8]See Appendix A for more details on wage, employment, and occupation definitions. The three-digit



(a) Employment Polarization (b) Wage Polarization

Fig. 3: Job and Wage Polarization, 1980–2010.

Source: U.S. Census (5%), extends Autor and Dorn (2013), which ends in 2005. Occupations are ranked by their 1980 mean wage for 11 one-digit groups and smoothed across 322 three-digit groups, separately. The widths of the bars are the employment share (in percent) in 1980. The y-axis measures the 30-year changes, of which units are in percentage points per percentile in panel (a). More details are in Appendix A.

In Figure 3(a), the percentage point change of a group's employment share is represented by the area of the bar,[9] while the height of each bar in Panel (b) measures the change in its mean wage. Except for sales and technicians, the patterns of polarization emphasized by Acemoglu and Autor (2011) and Autor and Dorn (2013) are intact.[10]

Polarization and structural change We now ask whether polarization and structural change are interrelated. In Figure 4, we plot the same data but along two different dimensions. In Panel (a), occupations are ordered along the x-axis in the same way as we did in Figure 3. For each occupation, we compute the employment share of manufacturing in 1980 and 2010, and plot the difference.[11] The bars measure the percentage point change in the share of manufacturing employment within each COC occupation group, and the dots the smoothed percentage point change for each skill percentile. The entire plot is negative, which represents structural change from manufacturing to services. More important, manufacturing shrank the most in the middle (again, except technicians).[12]

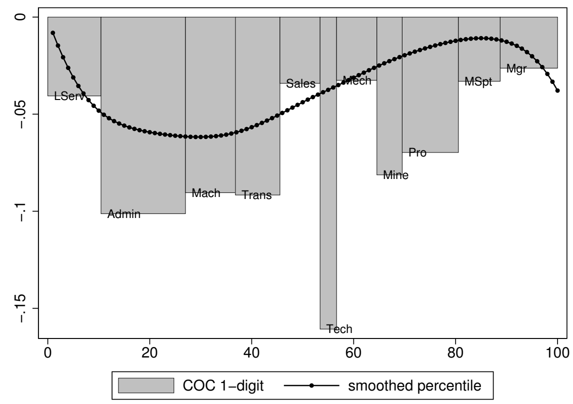
occupations in each group do not necessarily correspond to those used for the smooth graphs by percentile.

[9]By construction, the area of all bars must add up to 0. The smoothed graph should also integrate to 0 in theory, but does not due to the locally weighted regression errors.

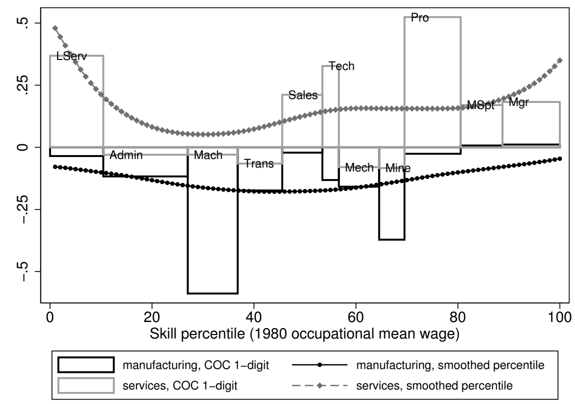
[10]The exact numbers behind these graphs are summarized in Appendix A Table 5, which also form the basis for our calibration in Section 4.

[11]Appendix Figure 20(a) shows the share of manufacturing employment within each occupation in 1980, which shows manufacturing's reliance on middle-skill jobs.

[12]Technicians include software engineers and programmers, paralegals, and health technicians, which grew rapidly during this time period. Indeed, many of the smooth graphs are flatten due to occupations in this group. However, they comprise a very small fraction of the U.S. economy throughout the sample period.



(a) Change in Manufacturing Employment Share



(b) Employment Polarization by Sector

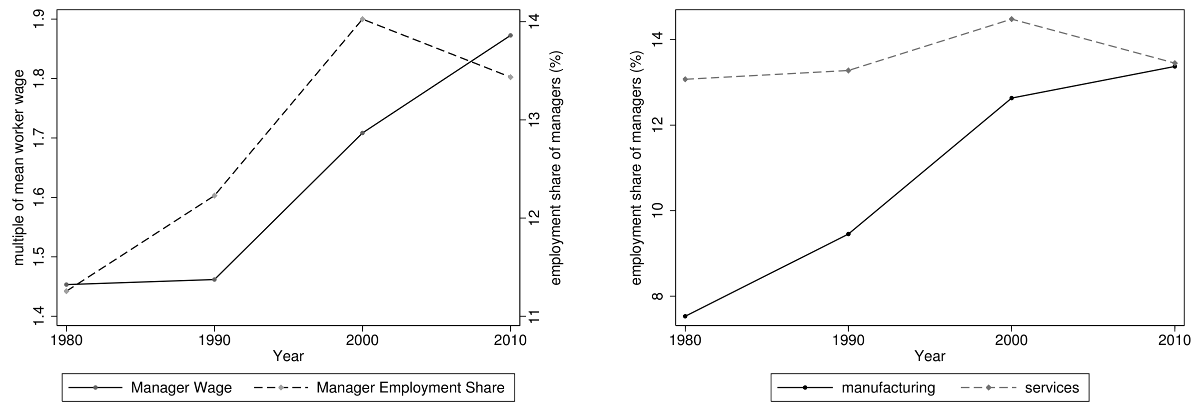
Fig. 4: Polarization and Structural Change, 1980–2010.
Source: U.S. Census (5%). Left: Percentage point change in manufacturing employment share within occupation. Right: Percentage point change in occupation employment by sector. Occupations are ranked by their 1980 mean wage for 11 one-digit groups and smoothed across 322 three-digit groups, separately. The x-axis units are 1980 employment shares (in percent). The y-axis measures the 30-year change, of which units are in percentage points per percentile in Panel (b). Further details are in Appendix A.

In Panel (b) we plot the same changes as in Figure 3(a), but separate manufacturing (in dark) and services (in light).[13] Manufacturing lost employment across all jobs (except managers), which again is structural change. This loss is more pronounced in the middle, especially machine operators and miners. In contrast, services gained employment overall, but mostly among occupation-skill percentiles in the far left and right (e.g., low-skill services and professionals, respectively).

What is important is that polarization is observed within both sectors, pointing toward the presence of task-specific forces: With sector-specific forces alone, we would expect employment shifts across occupations to be flatter in both panels. Moreover, as shown later in Figure 10, within-sector polarization was faster in manufacturing than in services: Between 1980 and 2010, the share of middle-skill jobs out of all manufacturing jobs fell by 13 percentage points, while the corresponding number in services was only 7 percentage points.

Vertical polarization In our model and quantitative analysis, we treat managers as a special occupation that organizes all other occupations. While many studies emphasize the organization of production (Garicano and Rossi-Hansberg, 2006), most focused on top CEO's of publicly traded companies (Tervio, 2008; Gabaix and Landier, 2008) or certain industries (Caliendo, Monte, and Rossi-Hansberg, 2015). We treat

[13]The area of all bars for one sector represents structural change, while adding them across both yields 0. Similarly, the integrals of the smoothed graphs should sum to 0, modulo locally weighted regression errors.



(a) Relative Employment and Wages

(b) Manager Employment Share

Fig. 5: Managers vs Workers

Source: U.S. Census (5%). Left: Relative wage and employment share of managers in aggregate. Right: Employment share of managers within manufacturing and services. See Appendix A for how we define managers in the census and Figure 19 for a detailed breakdown of the manager group.

managers as a broader group including CEOs, middle managers, and the self-employed, and also connect them to a notion of establishment.[14]

Previous papers have shown top CEO wages rising astronomically compared to the median worker's, and Figure 5(a) shows that even with our broader definition, the "manager wage premium" over all other workers rose from 45 percent in 1980 to 90 percent in 2010. At the same time, the employment share of managers also rose from 11 to 13.5 percent, although there is a small drop between 2000 and 2010.[15] We refer to this rise of managers relative to workers as a whole, in terms of both employment and wages, as "vertical polarization," to distinguish from the horizontal polarization across workers discussed above.

More important for us, vertical polarization was faster in manufacturing. Figure 5(b) shows that managers' employment share rose mostly in manufacturing and barely at all in services, and Appendix Figure 21 shows that the manager-worker mean wage ratio grew relatively more in manufacturing as well. This again suggests task- or occupation-specific forces, since sector-specific forces would not create such differences across sectors.[16]

[14]The inclusion of the self-employed in our definition of managers affects levels but not trends. That is, our quantitative results are robust to how the self-employed are classified—for example, to dropping all self-employed who are not managers. See Appendix Figure 19 and its corresponding text for detail.

[15]A separate analysis of the American Community Survey, not shown here, shows that managers' employment share continued to rise up to 14.5 percent by 2005, but then dropped by more than a percentage point, especially since the Great Recession.

[16]In Appendix Figure 22(a), we instead plot the manufacturing employment share among managers and workers, which shows that structural change was much more prevalent among workers than managers. This

We now present a model that can explain all the phenomena in this section with a single force: task-specific technological progress among middle-skill worker occupations.

2 Model

There is a continuum of individuals endowed with two types of skill, $(h, z) \in \mathcal{H} \times \mathcal{Z} \subset \mathbb{R}^2_+$. Worker human capital, h, is used to produce worker tasks. Managerial talent, z, is a skill for organizing tasks. Without loss of generality, we assume that the mass of individuals is 1, with associated cumulative distribution function $\mu(h, z)$.

There are two sectors $i \in \{m, s\}$.[17] In each sector, goods are produced by teams. A team is led by a manager who uses his managerial skill and physical capital to organize a continuum of worker tasks $j \in \mathcal{J} = [0, J]$, where J is finite.

We will refer to the manager's job as "task z," which is vertically differentiated (in a hierarchical sense) from tasks $j \in \mathcal{J}$, which are horizontally differentiated among workers. Each worker task requires both physical and worker human capital, and their allocations are decided by the manager. Specifically, we assume that

$$y_i(z) = \left[\eta_i^{\frac{1}{\omega}} x_z(z)^{\frac{\omega-1}{\omega}} + (1-\eta_i)^{\frac{1}{\omega}} x_h(z)^{\frac{\omega-1}{\omega}}\right]^{\frac{\omega}{\omega-1}}, \tag{1a}$$

$$x_z(z) = M(z) k_{iz}(z)^{\alpha} z^{1-\alpha}, \qquad x_h(z) = \left[\int_{j=0}^{J} \nu_i(j)^{\frac{1}{\sigma}} \tau_i(j;z)^{\frac{\sigma-1}{\sigma}} dj\right]^{\frac{\sigma}{\sigma-1}}, \tag{1b}$$

$$\tau_i(j;z) = M(j) k_{ih}(j;z)^{\alpha} \left[\int_{\mathrm{h}_i(j;z)} b(h,j) d\mu\right]^{1-\alpha}, \tag{1c}$$

with $\{\nu_i(j), \eta_i\} \in (0,1)$ for all $i \in \{m, s\}$ and $j \in \mathcal{J}^z \equiv \mathcal{J} \cup \{z\}$, and $\int_j \nu_i(j) dj = 1$.[18] It is important to note that sectors are different in terms of how intensively they use each task in production—i.e., ν and η have subscript i. The quantity $\tau_i(j; z)$ is the amount of task j output produced for a manager of skill z in sector i. This manager uses physical capital $k_{iz}(z)$ for himself, and allocates capital $k_{ih}(j; z)$ and a set of workers $\mathrm{h}_i(j; z)$ to task j. The function $b(h, j)$ is the productivity of a worker with human capital h assigned to task j.

Assumption 1 (Log-supermodularity) *The function $b : \mathcal{H} \times \mathcal{J} \mapsto \mathbb{R}_+$ is strictly positive, twice-differentiable, and log-supermodular. That is, for all $h' > h$ and $j' > j$:*

$$\log b(h', j') + \log b(h, j) > \log b(h', j) + \log b(h, j'). \tag{2}$$

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is further evidence against sector-specific forces.

[17] In our application, the two sectors indexed by m and s stand for "manufacturing" and "services," respectively. However, the analysis can be extended to any finite number N of sectors.

[18] A useful mnemonic is index i for industry (sector) and j for job (task).

Assumption 1 ensures that high-h workers sort into high-j tasks in equilibrium. Integrating $b(h,j)$ over h of the workers in the set $\mathrm{h}_i(j;z)$ yields the total productivity of all workers assigned to task j by a manager of skill z in sector i.

The elasticity parameter σ captures substitutability among tasks, while ω captures the elasticity between the composite worker output x_h and managerial output x_z. The $M(j)$'s, $j \in \mathcal{J}^z$, are task-specific TFP's, which are sector-neutral.

Let $\mathcal{Z}_i$ denote the set of individuals working as managers in sector i. Aggregating over the output from all managers in sector i yields sectoral output

$$Y_i = A_i \int_{\mathcal{Z}_i} y_i(z) d\mu, \quad i \in \{m, s\}, \tag{3}$$

where A_i is an exogenous, sector-level productivity parameter. Final goods are produced by combining output from both sectors according to a CES aggregator:

$$Y = \left[\gamma_m^{\frac{1}{\epsilon}} Y_m^{\frac{\epsilon-1}{\epsilon}} + \gamma_s^{\frac{1}{\epsilon}} Y_s^{\frac{\epsilon-1}{\epsilon}}\right]^{\frac{\epsilon}{\epsilon-1}}, \tag{4}$$

where $\gamma_m + \gamma_s = 1$. We will assume $\epsilon < 1$.[19]

The setup of our model is schematically visualized in Figure 6.[20]

2.1 Planner's Problem

Since our model has no frictions, the planner's allocation coincides with a competitive equilibrium allocation coincide.[21] A planner allocates aggregate capital K and all individuals into sectors $i \in \{m, s\}$ and tasks $j \in \mathcal{J}^z$. Formally, define $h_i(j;z)$ as the amount of human capital the planner allocates to task j in sector i under a manager with z. Also define $l_{ih}(s,j)$ as the number of individuals with skill $s = (h,z)$ that the planner assigns to task j, and $l_{iz}(s)$ the number of individuals with skill s the planner assigns as managers, in sector i. Then the planner's problem is to choose factor allocation rules $\{k_{iz}(z), k_{ih}(j;z), h_i(j;z)\}$ and assignment rules $\{l_{ih}(s,j), l_{iz}(s)\}$ to maximize current output (4) subject to

$$Y_i = A_i \int y_i(z) l_{iz}(s) ds, \quad \forall i \in \{m, s\},$$

[19]The estimated ϵ between the manufacturing and the service sectors (broadly defined) is close to 0, as we show in section 4.2.

[20]Except Costinot and Vogel (2010), most task-based models assume that either tasks or skills are a continuum, but not both (Acemoglu and Autor, 2011; Stokey, 2016). The worker side of our model is similar to theirs, but includes capital as a production factor and is extended to multiple sectors. In contrast to all existing models, we add two-dimensional skills and consider managers as a special occupation, which generates additional insights both theoretically and quantitatively.

[21]Nancy Stokey shared with us the insight that solving for the competitive equilibrium is more intuitive than solving for the planner's problem. While we agree in principle, the equilibrium approach necessitates introducing several implicit price functions due to our model's nested structure of production and inclusion of capital, as shown in Appendix B. In the main text we solve the planner's problem and present how to solve for prices in Section 2.4.

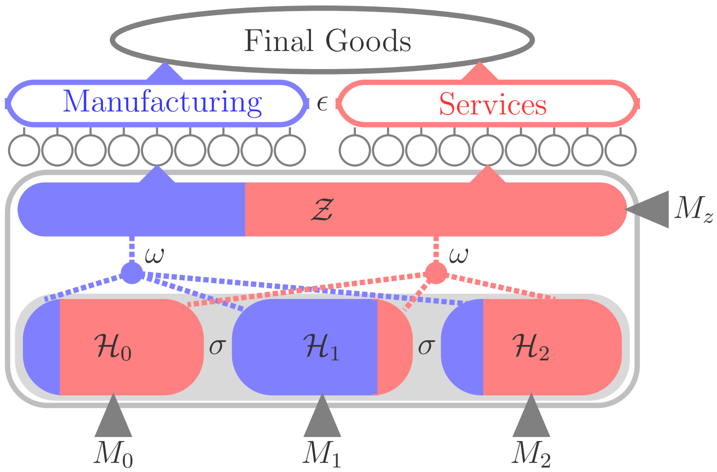


Fig. 6: Model

Individuals sort into managers and workers according to their skill (z, h), and workers further sort into tasks. While the model has a continuum of skills and tasks, in the figure we depict the latter as three discrete groups. Tasks are complementary with each other according to σ, and workers' output is complementary with managers' according to ω. Each team is led by a manager, and the collection of team output is sectoral output. The sectoral outputs form final output according to an elasticity parameter ϵ. The shaded areas show that sectors differ in how intensively they use each task in production.

$$K = K_m + K_s \equiv \sum_{i\in\{m,s\}} \int \left\{ \left[k_{iz}(z) + \int_j k_{ih}(j; z)dj \right] \cdot l_{iz}(s) \right\} ds \tag{5}$$

$$H_i(j) \equiv \int h_i(j; z) \cdot l_{iz}(s)ds = \int b(h, j)l_{ih}(s, j)ds \quad \forall i \text{ and } \forall j \in \mathcal{J},$$

$$d\mu(s) = \sum_{i\in\{m,s\}} \left[\int l_{ih}(s, j)dj + l_{iz}(s) \right] ds, \quad \forall s \in \mathcal{H} \times \mathcal{Z}, \tag{6}$$

where K_i is the capital allocated to sector i, and $H_i(j)$ the total productivity of workers allocated to task j in sector i.

For uniqueness of a solution, we assume that

Assumption 2 *The domain of skills $\mathcal{H} \times \mathcal{Z} = [0, h_M] \times [0, \infty)$, where $h_M < \infty$ is the upper bound of h. The measure $\mu(h, z)$ is continuously differentiable on $\mathcal{H} \times \mathcal{Z}$.*

Assumption 2 implies that we can write

$$\mu(\tilde{h}, \tilde{z}) = \int^{\tilde{h}} \int^{\tilde{z}} dF(z|h)dG(h) = \int^{\tilde{h}} \left[\int^{\tilde{z}} f(z|h)dz \right] g(h)dh,$$

where (G, g), the marginal c.d.f. and p.d.f. of h, and (F, f), the c.d.f and p.d.f. of z conditional on h, are continuous.

The optimal factor allocation rules across managers, $\{k_{iz}(z), k_{ih}(j; z), h_i(j; z)\}$, are straightforward: They must equalize marginal products across managers with different z's. Since we assume a constant returns to scale technology at the level of managers, we can aggregate over managers in (1) to write sectoral output as

$$Y_i = A_i \left[\eta_i^{\frac{1}{\omega}} X_{iz}^{\frac{\omega-1}{\omega}} + (1-\eta_i)^{\frac{1}{\omega}} X_{ih}^{\frac{\omega-1}{\omega}} \right]^{\frac{\omega}{\omega-1}}, \tag{7a}$$

$$X_{iz} = M(z) K_{iz}^{\alpha} Z_i^{1-\alpha}, \qquad X_{ih} = \left[\int_j \nu_i(j)^{\frac{1}{\sigma}} T_i(j)^{\frac{\sigma-1}{\sigma}} dj \right]^{\frac{\sigma}{\sigma-1}}, \tag{7b}$$

where K_{iz} is the total amount of capital allocated to managers and $Z_i \equiv \int z l_{iz}(s) ds$. Similarly, the sectoral task composite X_{ih} combines sectoral task aggregates

$$T_i(j) = M(j) K_{ih}(j)^{\alpha} H_i(j)^{1-\alpha}, \tag{8}$$

where $K_{ih}(j)$ is the total amount of capital allocated to task j in sector i.

In the remainder of this section, we characterize the solution to the planner's problem in the following order: (i) optimal physical capital allocations across tasks within a sector; (ii) optimal worker assignment across tasks within a sector; (iii) optimal allocation of managers vs. workers within a sector. We then solve for the within-sector solution in Section 2.2, which allows us to express the sectoral production function (3) only in terms of the optimal assignment rules and sectoral aggregates. Given this, we show in Section 2.3 that the two-sector equilibrium is unique, which enables comparative statics in Section 3.

Capital allocation within sectors The planner equalizes the marginal product of capital across tasks. Given the technologies in (7)-(8), this means that all capital decisions can be expressed as a linear function of the capital used in task 0. Specifically,

$$\pi_{ih}(j) \equiv \frac{K_{ih}(j)}{K_{ih}(0)} = \left[\frac{\nu_i(j)}{\nu_i(0)} \right]^{\frac{1}{\sigma}} \cdot \left[\frac{T_i(j)}{T_i(0)} \right]^{\frac{\sigma-1}{\sigma}}, \tag{9}$$

with which we can express the worker task composite X_{ih} in (8) as

$$X_{ih} = \nu_i(0)^{\frac{1}{\sigma-1}} \Pi_{ih}^{\frac{\sigma}{\sigma-1}} T_{i0}, \quad \text{where } \Pi_{ih} \equiv \int_j \pi_{ih}(j) dj. \tag{10}$$

Of course, marginal products must also be equalized across the managerial task and the rest: Using (10) we can define

$$\pi_{iz} \equiv \frac{K_{iz}}{K_{ih}(0)} = \left(\frac{\eta_i}{1-\eta_i} \right)^{\frac{1}{\omega}} \cdot \left(\frac{X_{iz}}{X_{ih}} \right)^{\frac{\omega-1}{\omega}} \cdot \Pi_{ih}, \tag{11}$$

which does not vary with j. Equations (9) and (11) subsume the capital allocation decisions into the labor allocation rules through $\pi_{ih}(j)$ and π_{iz}.

Sorting workers across tasks within sectors Since we assume $b(h, j)$ is strictly log-supermodular, Assumptions 1-2 imply that there exists a continuous assignment function $\hat{j} : [0, h_M] \mapsto \mathcal{J}$ s.t. $\hat{j}'(h) > 0$, and $\hat{j}(0) = 0$, $\hat{j}(h_M) = J$.[22] That is, there is

[22]For a more formal proof, refer to Lemma 1 in Costinot and Vogel (2010).

positive sorting of workers into tasks, and workers of skill h are assigned to job $\hat{j}(h)$. Since $\hat{j}'(h) > 0$, we can also define its inverse $\hat{h} : \mathcal{J} \mapsto [0, h_M]$.

It should be clear that $\hat{j}(h)$ and $\hat{h}(j)$ are identical across sectors, and hence not indexed by i. Otherwise, the planner would be able to reallocate h across sectors and increase output. So the planner's problem of choosing $l_{ih}(s, j)$ has two parts: One of choosing $l_h(s, j)$, the number of individuals with skill s assigned to task j *regardless of sector*, and the other of choosing $q_{ih}(j)$, the fraction of task j workers allocated to sector i, which satisfies $\sum_{i\in\{m,s\}} q_{ih}(j) = 1$. That is, we can write $l_{ih}(s, j) = q_{ih}(j) \cdot l_h(s, j)$.

Sorting managers vs. workers within a sector Standard Roy selection with two-dimensional heterogeneity implies a cutoff rule $\tilde{z}(h)$ such that for every h, individuals with z above $\tilde{z}(h)$ become managers and the rest become workers. Since we know that workers sort positively into j, we also define $\hat{z}(j) = \tilde{z}(\hat{h}(j))$ for all $j \in \mathcal{J}$. As with workers, the manager selection rule must be identical across sectors.

Similar to Lucas (1978), $\hat{z}(j)$ is chosen so that the marginal product of the threshold manager is equalized between task z and j, so

$$\hat{z}(j)/b(\hat{h}(j), j) = \pi_{ih}(j) Z_i / \pi_{iz} H_i(j), \quad \forall j \in \mathcal{J}. \tag{12}$$

Without loss of generality, we normalize $b(0, 0) = 1$ to obtain

$$\hat{z} \equiv \hat{z}(0) = \tilde{z}(0) = Z_i / \pi_{iz} H_i(0) \tag{13}$$

which is the worker counterpart of (11): the total productivity of managers in sector i is normalized in terms of the task-0 productivity, $H_i(0)$. In the next subsection, we normalize all other workers' productivities by $H_i(0)$ as well.

2.2 Within-Sector Solution

First consider the rule $l_{iz}(s)$. Since the rule $\tilde{z}(h) = \hat{z}(\hat{j}(h))$ does not depend on sector,

$$\int l_{iz}(s) ds = q_{iz} \int [1 - F(\tilde{z}(h)|h)] \, dG(h), \tag{14}$$

where q_{iz} is a sectoral weight that satisfies $\sum_{i\in\{m,s\}} q_{iz} = 1$. Note that any solution that satisfies (14) such that $l_{iz}(s) = 0$ iff $z \leq \hat{z}(j)$ is optimal. Hence, even if the planner's choices of $\hat{z}(j)$ and q_{iz} are unique, the rule $l_{iz}(s)$ is not: The planner does not care how managers are allocated between sectors for any particular $s \in \mathcal{S}$, and only about how the total Z is divided between sectors, where

$$Z \equiv \int \int_{\tilde{z}(h)}^{\infty} z dF(z|h) dG(h) = \sum_{i\in\{m,s\}} q_{iz} Z_i.$$

Next consider the planner's choice of $l_h(s, j)$. The characterization is similar to Costinot and Vogel (2010) and summarized in Lemma 1.

Lemma 1 *Define*

$$B_j(j;\hat{h}) \equiv \exp\left[\int_0^j \frac{\partial \log b(\hat{h}(j'),j')}{\partial j'}dj'\right]. \tag{15}$$

At the optimum, the productivity of all workers assigned to task j can be expressed as

$$H(j) = b(\hat{h}(j),j)\cdot F(\hat{z}(j)|\hat{h}(j))g(\hat{h}(j))\cdot \hat{h}'(j), \quad \forall j \in \mathcal{J}, \tag{16}$$

and their ratios across tasks in sector i must satisfy

$$q_{ih}(j)H(j) = H_i(j) = \pi_{ih}(j)H_i(0)B_j(j;\hat{h}), \tag{17}$$

Proof *See Appendix C.1.* ∎

The lemma expresses all other worker allocation decisions in terms of $H_i(0)$—just as we could normalize all other capital allocation by $K_i(0)$ in (9) and (11). Equation (16) simply states that total worker productivity in task j is the product of the infinitesimal mass of individuals assigned to task j, times their effective productivity. Equation (17) is the counterpart of (9): the marginal products of labor are equated at every point along $\mathcal{J}$. Similarly, all manager allocations can also be normalized by $\hat{z}$.

Corollary 1 *Define the counterpart of B_j in (15):*

$$B_h(h;\hat{j}) \equiv \exp\left[\int_0^h \frac{\partial \log b(h',\hat{j}(h'))}{\partial h'}dh'\right]. \tag{18}$$

At the optimum, the productivity of the cutoff rules $\hat{z}(j)$ and $\tilde{z}(h)$ can be expressed as

$$\hat{z}(j) = \hat{z}\cdot b(\hat{h}(j),j)/B_j(j;\hat{h}) = \hat{z}\cdot B_h(\hat{h}(j);\hat{j}) \quad \Leftrightarrow \quad \tilde{z}(h) = \hat{z}\cdot B_h(h;\hat{j}). \tag{19}$$

Proof *See Appendix C.1.* ∎

The corollary makes all manager cutoff rules except $\hat{z}$ (the threshold at $h=0$) redundant, since they can be expressed only in terms of $\hat{z}$ in (13) and $\hat{h}(j)$. In what follows, we suppress the dependence of B_j and B_h on $\hat{h}$ and $\hat{j}$ unless necessary.

So far we have normalized all allocations only in terms of the task-0 capital and worker input, $K_i(0)$ and $H_i(0)$. Now we show that given a sectoral allocation rule for each task, $q_{ih}(j)$ and q_{iz}, the within-sector equilibrium is unique if

Assumption 3 *For all $h \in \mathcal{H}$,*

1. *$\partial b(h,j)/\partial h > 0$ for all $j \in \mathcal{J}$, and*
2. *$g'(h) \leq 0$ and $f'(z|h) \leq 0$ for all $z \in \mathcal{Z}$.*

Assumption 3.1 captures the notion that higher-h workers perform better in any task; in particular, under this assumption, $\tilde{z}(h)$ in (19) is a strictly increasing function. Assumption 3.2 means that there are fewer people at higher levels of skill, which is a common assumption and also consistent with empirical evidence.

Under these assumptions, the within-sector allocation is completely independent of sectoral capital and labor, which admits a sectoral production function in which sectoral TFP is solely determined by the optimal allocation rules.

Proposition 1 *Suppose $q_{ih}(j)$ and q_{iz} are given. Under Assumptions 1-3, the within-sector solution to the planner's problem $\left\{[\hat{h}(j)]_{j=0}^{J}, \hat{z}\right\}$ exists, is unique, and is independent of sectoral aggregates K_i and L_i.*

Proof *See Appendix C.2.* ∎

Corollary 2 *At the planner's optimum, sectoral output can be expressed as*

$$Y_i = \Phi_i \cdot K_i^{\alpha} L_i^{1-\alpha}, \quad \textit{where } \Phi_i \equiv M(0) \cdot \psi_i \cdot \Pi_{ih}^{\frac{\sigma-\omega}{(\sigma-1)(1-\omega)}} \Pi_{K_i}^{\frac{\omega}{\omega-1}-\alpha} \Pi_{L_i}^{\alpha-1} \tag{20}$$

is the sectoral TFP, endogenously determined by the optimal allocation rules. Sectoral TFP can be decomposed into 3 parts:

1. *$M(0)$, which is common across both sectors and exogenous;*
2. *$\psi_i \equiv A_i(1-\eta_i)^{\frac{1}{\omega-1}} \nu_{i0}^{\frac{1}{\sigma-1}}$, which is also exogenous but sector-specific;*
3. *the part determined by $(\Pi_{ih}, \Pi_{K_i}, \Pi_{L_i})$, which is sector-specific and endogenously determined by the allocation rules $\hat{h}(j)$ and $\hat{z}$, where*

$$\Pi_{K_i} \equiv \Pi_{ih} + \pi_{iz} = K_i/K_i(0) \textit{ and } \Pi_{L_i} \equiv \Pi_{il} + (\hat{z}/\bar{z})\pi_{iz} = L_i/H_i(0) \tag{21}$$

are the total amounts of capital and labor in sector i in units of the task-0 capital and labor allocated to sector i, respectively, and $\Pi_{il} \equiv \int \left[\pi_{ih}(j)/B_h(\hat{h}(j))\right] dj$.

Proof *See Appendix C.2.* ∎

Sectoral TFP's are independent of sectoral capital and labor shares because the rules $\hat{h}(j)$ and $\hat{z}$ depend only on the relative masses of individuals across tasks *within* a sector, and not on the employment shares *across* sectors. In fact, it is the sectoral TFP's that determine sectoral input shares. Since sectors only differ in how intensively they use each task, employment shares are determined so that the marginal products of capital and labor are equalized across sectors:

$$\kappa \equiv \frac{K_s}{K_m} = \frac{L_s}{L_m} = \left(\frac{\gamma_s}{\gamma_m}\right)^{\frac{1}{\epsilon}} \left(\frac{Y_s}{Y_m}\right)^{\frac{\epsilon-1}{\epsilon}} = \frac{\gamma_s}{\gamma_m} \cdot \left(\frac{\Phi_s}{\Phi_m}\right)^{\epsilon-1} \tag{22}$$

where κ is the capital input ratio between sector s and m. Hence relative employment between the two sectors is completely determined by their endogenous TFP ratio. Since the Φ_i's are just functions of $\hat{h}(j)$ and $\hat{z}$, so are κ and sectoral employment shares L_i:

$$L_m = 1/(1+\kappa),\ L_s = \kappa/(1+\kappa). \tag{23}$$

Consequently, the aggregate levels of K or L have no impact whatsoever on the assignment rules and employment shares.

2.3 Existence and Uniqueness of Full Solution

Since the planner's solution coincides with an equilibrium in our economy, existence and uniqueness of an equilibrium is equivalent to a unique solution to the planner's problem. As a final step, the planner needs to ensure that the within-sector allocations are consistent with the between-sector allocations. That is, the weights used to split the distribution μ between sectors, $q_{ih}(j)$ and q_{iz}, must be consistent with (22). These are equivalent to the the labor market clearing conditions for an equilibrium.

For ease of notation, let $q_h(j)$ and q_z denote the service share of employment in tasks j and z, respectively; so $q_{mh}(j) = 1 - q_h(j)$ and $q_{mz} = 1 - q_z$. Since $\hat{h}(j)$ and $\hat{z}$ must be equal across sectors, we can use the within-sector solutions from Proposition 1 to find the $q_h(j)$ and q_z that ensure this. The proposition already showed that the within-sector solution is unique, but for uniqueness of the full solution we need additional assumptions on μ and $b(h,j)$ that will serve as sufficient conditions:

Assumption 4 *For all $(h,z) \in \mathcal{H} \times \mathcal{Z}$,*

1. $F(z|h)/\left[1 - F(z|h)\right] \leq z/\left[\int_z^\infty z' f(z'|h)dz'\right]$, *and*
2. $zf(z|h)/F(z|h) \geq (1-\alpha)(1-\omega)$.

Assumption 4 means that the conditional distribution of z is declining but not too much, in the sense that it still has fat tails beyond any value of h.

Assumption 5 *For all $(h,j) \in \mathcal{H} \times \mathcal{J}$, $0 < \partial^2 \log b(h,j)/\partial h \partial j < \varepsilon$ for all $\varepsilon > 0$.*

That the cross partial is larger than 0 is already in Assumption 1. Assumption 5 just means that there is just enough log-supermodularity so that workers positively sort into tasks. While this may be a strong restriction, it also means that our results will hold for any equilibria in the vicinity of no sorting.

Theorem 1 *Under Assumptions 1-5, the solution to the planner's problem, $[\hat{h}(j), q_h(j)]_{j=0}^J$ and $(\hat{z}, q_z)$, exists and is unique.*

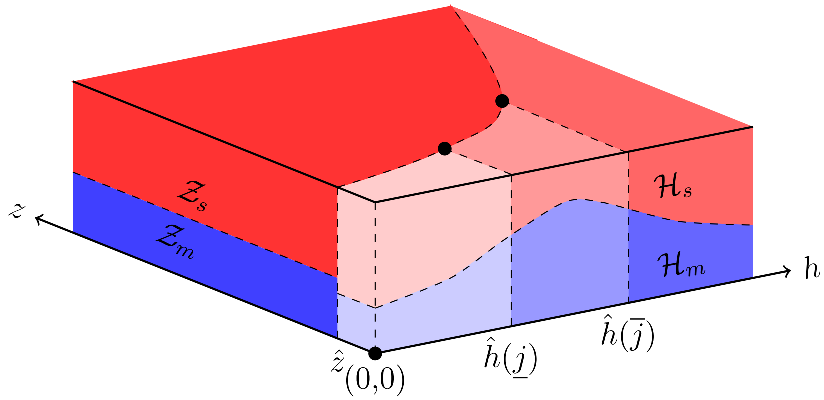


Fig. 7: Equilibrium

Proof *See Appendix C.3.* ∎

For illustrative purposes, the equilibrium skill allocation with a uniform $\mu(z, h)$ is depicted in Figure 7. Individuals in $\mathcal{Z}$ are managers, and those in $\mathcal{H}$ workers, while the subscripts s and m denote services and manufacturing. Workers sort into tasks indexed by j according to $\hat{h}(j)$. The different masses of sectoral employment across tasks are due to the task intensity parameters $\nu_i(j)$ and η_i.

2.4 Equilibrium Wages and Prices

The solution $\hat{h}(j)$ and $\hat{z}$ give all the information needed to derive equilibrium prices (which are unique). The price of the final good can be normalized to 1:

$$P = 1 = \left[\gamma_m p_m^{1-\epsilon} + \gamma_s p_s^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}, \qquad p_i = [Y_i/\gamma_i Y]^{-\frac{1}{\epsilon}} \tag{24}$$

and sectoral output prices p_i are obtained by plugging in the sectoral production function (20). Let R denote the rental rate of capital and $w_h(h)$ the wage of a worker with skill h. Since capital and labor input ratios are equalized across sectors, $w_h(0)$ can be found from either sector:

$$w_h(0) = \frac{1-\alpha}{\alpha} \cdot \frac{RK_i(0)}{H_i(0) \cdot b(0,0)} = \frac{1-\alpha}{\alpha} \cdot \frac{\Pi_{L_i}}{\Pi_{K_i}} \cdot RK,$$

where the second equality follows from (20)-(22), and also from our normalization of both $b(0, 0)$ and the population size to 1. Similarly, all workers earn their marginal product, so we can write

$$w_h(h) = w_h(0) \cdot B_h(h). \tag{25}$$

Assumption 3.1 implies that $w(h)$ is strictly increasing in h. For all $h \in \mathcal{H}$, threshold managers with skill $z = \tilde{z}(h)$ are indifferent between becoming a worker or manager, so we can determine a managerial wage rate or rental rate of z (i.e., w_z) that satisfies

$$w_z \tilde{z}(h) = w_h(h) \quad \Rightarrow \quad w_z = w_h(0)/\hat{z}, \tag{26}$$

and (19) and (25) guarantee that the first equality holds for all h.

3 Comparative Statics

We now explore the implications of changes in task-specific TFP or $M(j)$.[23] In particular, we are interested in the effect of an increase in the TFP of middle-skill tasks. As shown in Section 5, these tasks are "routine jobs," and hence the productivity growth specific to them is called "routinization."

3.1 Wage and Job Polarization

First, we focus on the comparative statics for $\hat{h}(j)$ and $\hat{z}$ within a sector i, ignoring sectoral reallocation. Our exercise assumes that there is an increase in the exogenous productivity, $M(j)$, of middle-skill j-tasks.[24]

Proposition 2 (Routinization and Polarization) *Let $\mathcal{J}^1 \equiv (\underline{j}, \overline{j}) \subset \mathcal{J}$, where $0 < \underline{j} < \overline{j} < J$. Suppose $q_h(j)$ and q_z are held constant, and that $M(j)$ uniformly grows to $M^1(j) = M(j)e^{\tilde{m}}$ for all $j \in \mathcal{J}^1$, where $\tilde{m} > 0$. Then under Assumptions 1-4,*

1. *if $\sigma < 1$, there exists $j^* \in \mathcal{J}^1$ such that $\hat{h}^1(j) > \hat{h}(j)$ for all $j \in (0, j^*)$ and $\hat{h}^1(j) < \hat{h}(j)$ for all $j \in (j^*, J)$, and*
2. *if $\omega < \sigma < 1$ and Assumption 5 holds, there exists some $\varepsilon > 0$ s.t. $\tilde{z}^1(h) < \tilde{z}(h)$ for all $0 < \tilde{m} < \varepsilon$ and $h \in \mathcal{H}$.*

Proof *See Appendix C.4.* ∎

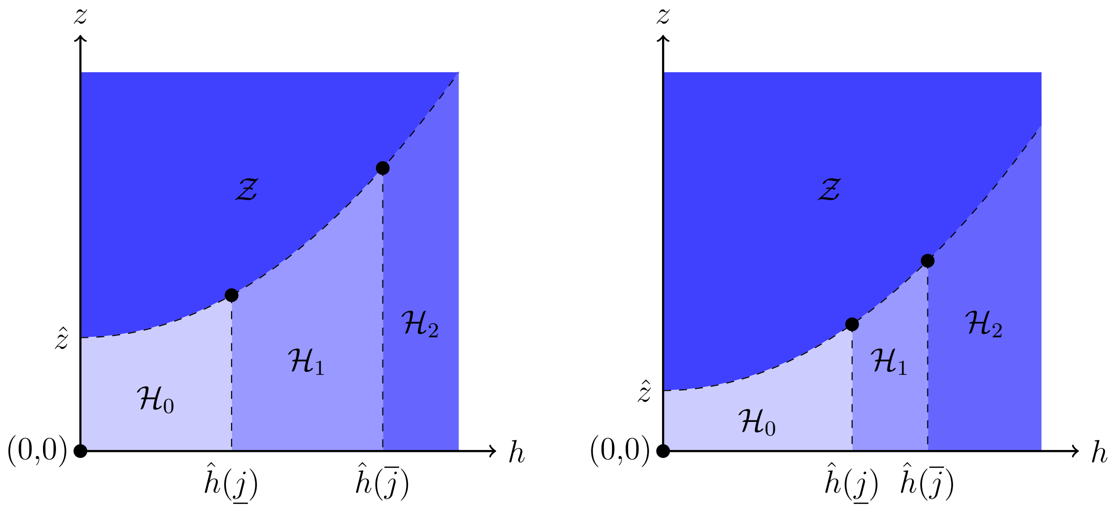
Part 1 implies that among worker tasks, capital and labor flow out of middle-skill tasks into the extremes (horizontal job polarization). The relative wages of the middle-skill tasks decline (horizontal wage polarization), since from (25),

$$\log\left[\frac{w^1(h)}{w^1(h^*)}\right] - \log\left[\frac{w(h)}{w(h^*)}\right] = \int_{h^*}^{h}\left[\frac{\partial \log b(h, \hat{j}^1(h))}{\partial h} - \frac{\partial \log b(h, \hat{j}(h))}{\partial h}\right] dh$$

is positive for all $j \neq j^* \in \mathcal{J}^1$. Part 2 implies that capital and labor flow into management from *all* worker tasks, and (26) means that each manager earns a higher

[23] A change in factor-neutral task-TFP, $M(j)$, is different from an increase in the amount of skill working in any given task. Since we model worker skill as human capital, the qualitative effect of a change in task-TFP is the same as if it were only capital-augmenting—e.g., a fall in the price of task-specific capital.

[24] This *within-sector* exercise is similar to Lemma 6 in Costinot and Vogel (2010), except that we have capital and, more important, the second dimension of skill that vertically separates managers from workers.



(a) Equilibrium with low $M(j),\ j \in \mathcal{J}^1$

(b) Equilibrium with high $M(j),\ j \in \mathcal{J}^1$

Fig. 8: Comparative Static, Within-Sector.

wage per managerial skill (vertical polarization). The within-sector comparative static for employment shares is depicted in Figure 8, and are consistent with the data we saw in Figures 3 and 5(a).

The mechanism for part 1 is the same as in Goos et al. (2014): when $\sigma < 1$, the exogenous rise in productivity causes factors to flow out to other tasks since tasks are complementary, and we get employment polarization. As in Costinot and Vogel (2010), this also leads to wage polarization in the presence of positive sorting. What is new in our model is that this happens even in the presence of the vertically differentiated management task, and that with stronger complementarity between workers and managers than among workers (i.e., $\omega < \sigma < 1$), similar forces drive vertical polarization in terms of both wages and employment. The most novel feature is the impact of such TSTC on sectoral allocations, which we now explain.

3.2 Structural Change

Previous models of structural change either rely on a non-homogeneous form of demand (rise in income shifting demand toward service products) or relative technology differences across sectors (rise in manufacturing productivity relative to services, combined with complementarity between the two, shifting production factors toward services). Our model is also technology driven, but structural change arises from a skill- and sector-neutral increase in task productivities that endogenously determines sectoral TFP's. Contrary to recent papers arguing that sectoral productivity differences generate broadly-measured skill premia or polarization (Buera et al., 2015; Bárány and

Siegel, 2018), we argue routinization explains not only job and wage polarization but also structural change. Moreover, those papers cannot address within-sector changes.

Decomposing polarization Define the "unnormalized" total worker productivity

$$V_{L_i} = (1-\eta_i)\nu_i(0)M(0)^{\sigma-1}\Pi_{L_i} = V_{il} + V_i(z), \quad \text{where } V_{il} = \int V_i(j)dj \tag{27}$$

and the weights $V_i(j)$ for each task $j \in \mathcal{J}^z$ are

$$V_i(j) = (1-\eta_i)\nu_i(j)\left[M(j)B_j(j)^{1-\alpha}\right]^{\sigma-1}/B_h(\hat{h}(j)) \tag{28a}$$

$$V_i(z) = \eta_i M(z)^{\omega-1} \cdot V_{ih}^{\frac{\sigma-\omega}{\sigma-1}} \cdot \hat{z}^{\alpha+\omega(1-\alpha)}/\bar{z}, \tag{28b}$$

$$V_{ih} \equiv \nu_i(0)M(0)^{\sigma-1}\Pi_{ih} = \int \left\{\nu_i(j) \cdot \left[\tilde{M}(j) \cdot B_j(j)^{1-\alpha}\right]^{\sigma-1}\right\} dj. \tag{29}$$

These are simply the marginal products of task j unnormalized by $H_i(0)$, so we know that taking the ratio between any pair yields the labor input ratio between the two tasks; (29) is the unnormalized counterpart of Π_{ih} in (10). So the definition of Π_{L_i} in (21) implies that the amount of labor (or worker human capital) in each task across both sectors can be expressed as

$$L(j) = \sum_{i\in\{m,s\}} \frac{L_i(j)}{L_i} \cdot L_i = \sum_{i\in\{m,s\}} \frac{V_i(j)}{V_{L_i}} \cdot L_i.$$

We consider the same exercise as in Proposition 2, that $M(j)$ grows to $M^1(j) = \exp\{\tilde{m}\}M(j)$ for $\tilde{m} > 0$ and all $j \in \mathcal{J}^1 \equiv (\underline{j}, \bar{j})$. Let Δ_X denote the log-derivative of X w.r.t. $\tilde{m}$, then

$$\begin{aligned} \Delta_{L(j)} &= \sum_{i\in\{m,s\}} \frac{L_i(j)}{L(j)} \cdot \left[\Delta_{V_i(j)} - \Delta_{V_{L_i}} + \Delta_{L_i}\right] \\ &= \sum_{i\in\{m,s\}} \frac{L_i(j)}{L(j)} \cdot \Bigg\{ \underbrace{\Delta_{V_i(j)} - \int_{\mathcal{J}^z} \left[\frac{V_i(j')}{V_{L_i}} \cdot \Delta_{V_i(j')}\right] dj'}_{W_{ij}} + \Delta_{L_i} \Bigg\}. \end{aligned} \tag{30}$$

A change in the $V_i(j)$'s occurs even holding L_i's constant, shifting the term W_{ij}. This leads to within-sector polarization, as we saw in Proposition 2. To compare the sectoral differences in its impact, we compare the $\Delta_{V_{il}}$ and $\Delta_{V_i(z)}$ across the two sectors, which represent, respectively, the change in workers and managers. Then we can sign $\Delta_{V_{L_i}}$, the change in sectoral employment shares which determines structural change.

Lemma 2 *Suppose $\omega < \sigma < 1$ in Proposition 2, so that we get both horizontal and vertical polarization within sectors. Then both horizontal and vertical polarization is faster in manufacturing in the sense that $\Delta_{V_{ml}} < \Delta_{V_{sl}} < 0$ and $\Delta_{V_m(z)} > \Delta_{V_s(z)} > 0$ if $L_m(j)/L_m > L_s(j)/L_s$.*

Proof *See Appendix C.5.* ∎

When the lemma holds, both horizontal and vertical polarization is faster in manufacturing, as we saw in the data in Figures 4 and 5. The assumptions in the lemma mean that the manufacturing sector is more dependent on middle-skill or routine jobs ($j \in \mathcal{J}^1$) relative to services, and services more on managers relative to manufacturing, which is evident in the data shown in Figures 5(b) and 20(a). Of course, these are assumptions on endogenous variables: Because we do not know the value of task productivities $M(j)$, $\hat{h}(j)$ and $\hat{z}$, this holds in general if there exists $\bar{\nu} \in (0,1)$ such that $\nu_m(j) - \nu_s(j) \geq \bar{\nu}$ for all $j \in \mathcal{J}^1$.

The lemma holds regardless of the value of $q_h(0)$, which determines the between-sector equilibrium. So W_{ij} in (30) gives the equilibrium change in within-sector employment shares coming only from a change in the selection rules $\hat{h}(j)$ and $\hat{z}$. Clearly, a change in the rules will also alter the last term, Δ_{L_i}, which captures between-sector allocation, or structural change. Lemma 3 summarizes when structural change is in the direction of shifting capital and labor from manufacturing to services.

Lemma 3 (Structural Change) *Suppose $\omega < \sigma < 1$ in Proposition 2, so that routinization causes both horizontal and vertical polarization within both sectors. If $\epsilon < 1$, then there exists $(\bar{\nu}, \bar{\eta}) \in (0,1)$ such that for all $\nu_m(j) - \nu_s(j) \geq \bar{\nu}$, $j \in \mathcal{J}^1$, and $\eta_s - \eta_m \geq \bar{\eta}$,*

$$(\Delta_{V_{L_s}} = \Delta_{\Pi_{L_s}}) > (\Delta_{V_{L_m}} = \Delta_{\Pi_{L_m}}), \quad \Delta_{\Pi_{K_s}} > \Delta_{\Pi_{K_m}}, \quad \textit{and} \quad \Delta_{L_s} > 0$$

where the equalities follow from (27).

Proof *See Appendix C.5.* ∎

The additional assumption on η_i ensures that Lemma 2 holds even as the within-sector share of managers increase and routine jobs decrease, as the sectoral employment share of manufacturing declines.

It is subtle but structural change in fact has two parts. As a task becomes more productive than others, selection on skills ensures that less resources are allocated to it when we have complementarity across tasks (Proposition 2). If one sector uses the task that has become more productive more intensively, resources reallocate across sectors even holding fixed the sectoral allocation rule (Lemma 2). This is the first part.

The second part is that, as the manufacturing sector becomes more productive—endogenously because it uses the task that has become more productive more intensively than services—the equilibrium price of its output falls relative to services. The strength of this force is governed by the elasticity between manufacturing and service outputs, and with complementarity ($\epsilon < 1$), more resources are allocated to services. These two forces are formalized in Appendix C.5.

The appendix also formalizes that structural change depends differently on the productivity of capital and labor, as is apparent from (20)-(21). In contrast to Ngai and Pissarides (2007) and Goos et al. (2014), capital is homogenous in our model but labor is not, which is measured in two different types of skills. Since labor productivity is determined by sorting individuals of heterogeneous skills, how TSTC leads to changes in sectoral capital and labor input ratios depends not only on the task intensity of sectors, but also on changes in the selection rules.

Of course from (30), it is clear that structural change (change in Δ_{L_i}) also contributes to polarization. To see this more precisely, rewrite (30) using (23) as

$$\Delta_{L(j)} = \Delta_{V_i(j)} - \sum_{i \in \{m,s\}} \frac{L_i(j)}{L(j)} \cdot \Delta_{V_{L_i}} + \left[\frac{L_s(j)}{L(j)} \cdot L_m - \frac{L_m(j)}{L(j)} \cdot L_s \right] \Delta_\kappa \tag{31}$$

$$\Rightarrow \quad L(j) \left(\Delta_{L(j)} - \Delta_{V_i(j)} \right) = - \sum_{i \in \{m,s\}} L_i(j) \Delta_{V_{L_i}} + \left[\frac{V_s(j)}{V_s} - \frac{V_m(j)}{V_m} \right] L_m L_s \Delta_\kappa .$$

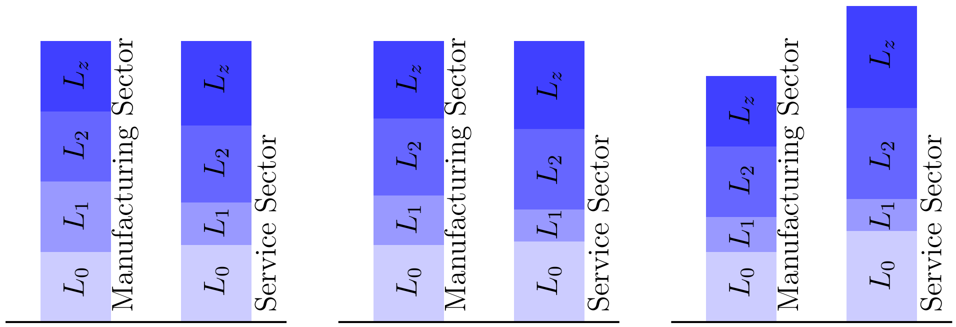
Lemma 4 *If Lemma 2 holds, then structural change also contributes to polarization.*

Proof *Under Lemma 2, the term in the square brackets in (31) is negative for $j \in \mathcal{J}^1$, and positive for $j = z$.* ∎

This is a compositional effect. If manufacturing is more reliant on middle-skill tasks and shrinks due to faster within-sector polarization, this leads to even more horizontal polarization in the aggregate economy. Also, because manufacturing is less reliant on managers, there is even more vertical polarization in the aggregate economy.

Lemmas 2-4 are depicted in the first 3 subplots of Figure 9. In Panel (a), manufacturing is depicted as having a higher share in intermediate tasks, and services in managers. As we move from (a) to (b), sectoral employment shares are held fixed, and intermediate tasks shrink in both sectors. The change in employment shares is larger in manufacturing due to Lemma 2. This leads to structural change in (c), according to Lemma 3. Because manufacturing uses intermediate tasks more intensively and managerial tasks less intensively than services, shrinking its size contributes to the horizontal and vertical polarization for the overall economy (not separately depicted).

In the model, TSTC—changes in $M(j)$—shifts relative employment shares as if the weights $\nu_i(j)$ and η_i were changing, so the two are not separately identified in our comparative statics. However, since the model is constructed so that the time-invariant weights capture an initial distribution of employment shares while TSTC



(a) Low $M(j)$, $j \in \mathcal{J}^1$ (b) High $M(j)$, $j \in \mathcal{J}^1$, κ fixed (c) High $M(j)$, $j \in \mathcal{J}^1$

Fig. 9: Comparative Statics, Across Sectors.

drive the changes over time, the assumptions we made in the lemmas are valid insofar as they hold throughout our observation period in the data.[25]

3.3 Polarization or Structural Change?

One may wonder whether it is not TSTC leading to structural change, but growth in sector-specific productivities leading to polarization, considering Lemma 4 in isolation.

One important fact is that, in our model, sector-specific productivity changes do not lead to polarization *within* sectors. To see this, consider a change in the manufacturing sector's exogenous productivity, A_m. As in Ngai and Pissarides (2007), a rise in A_m changes κ at a rate of $1-\epsilon > 0$; that is, manufacturing shrinks. It can easily be seen that none of the thresholds change, and hence neither do the Φ_i's (the endogenous sectoral TFP's). So polarization in the overall economy can only arise by the reallocation of labor across sectors *but while preserving their ratios within each sector*. To be precise, from (30),

$$\frac{d \log L(j)}{d \log A_m} = (1-\epsilon) \cdot \frac{d \log L(j)}{d \log \kappa} = (1-\epsilon) \left[\frac{L_s(j)}{L(j)} L_m - \frac{L_m(j)}{L(j)} L_s \right] < 0. \tag{32}$$

Note that $d \log L(j) / d \log \kappa$ is equal to the term in square brackets in (31), and negative for $j \in \mathcal{J}^1$. Hence, there is no within-sector polarization. The reason is that, in our model, tasks are aggregated up to sectoral output, not the other way around.

Even if one were to ignore within-sector polarization, our model—specifically (32)—provides an upper bound on how much job polarization can be accounted for by structural change. For example, in the data, the manufacturing employment share fell from

[25] When we calibrate the model to the 1980 data—for which we assume that $M(j) = M$ for all j—the calibration naturally admits that $\eta_m < \eta_s$ and $\nu_m(j) > \nu_s(j)$ for a wide range of middle-skill jobs. Furthermore, since occupational employment ratios between sectors are never flipped for most occupations up to 2010, the quantitative analysis is robust to the choice of normalization year.

33 percent to 19 percent between 1980 and 2010. If this were solely due to an exogenous change in A_m, denoting empirical values with hats:

$$\frac{d\log\hat{\kappa}}{d\log A_m} \approx \frac{0.14}{0.67} + \frac{0.14}{0.33} \approx 0.63.$$

Now denote all routine jobs as $j = 1$, then we can approximate

$$\frac{d\hat{L}_1}{d\log A_m} \approx 0.63 \cdot \left[\hat{L}_{s1}\hat{L}_m - \hat{L}_{m1}\hat{L}_s\right] = 0.63 \cdot \left[\frac{\hat{L}_{s1}}{\hat{L}_1} \cdot 0.33 - \frac{\hat{L}_{m1}}{\hat{L}_1} \cdot 0.67\right].$$

In Appendix Table 5, we measure the share of routine jobs in manufacturing and in services as a share of total employment—that is, $\hat{L}_{m1}$ and $\hat{L}_{s1}$—to be 26 and 33 percent in 1980, respectively. So

$$\frac{d\hat{L}_1}{d\log A_m} \approx 0.63\,[0.33 \cdot 0.33 - 0.26 \cdot 0.67] = -0.04,$$

which means that a change in A_m alone implies a 4-percentage-point drop in routine jobs from 1980 to 2010 in the overall economy. As shown in Table 5, the actual drop is 13 percentage points. In other words, an exogenous structural change can explain at best 30 percent of the polarization in the overall economy—and none within sectors.

4 Calibration

Our quantitative analysis will find out how much of the observed changes in employment and wage across occupations and sectors from 1980 to 2010 is explained by TSTC. Whenever possible, we fix parameters to their empirical counterparts, and separately estimate the aggregate technology (4) from the time series of sectoral price and output ratios. Then we choose most model parameters to fit the 1980 data exactly. The other parameters, which include the between-task elasticity parameters σ and ω, are calibrated to empirical trends from 1980 to 2010, without any sector-specific moments.

4.1 Parametrization

Discrete log-supermodularity In the quantitative analysis, we collapse the continuum of horizontally differentiated worker tasks into 10 groups, corresponding to the one-digit occupation groups in the census in Section 1 and Appendix A Table 5. (There is still only one management task.) The 10 worker occupation groups are further categorized into low/medium/high skill tasks, or manual/routine/abstract jobs, according to the mean wages of each occupation group.

To discretize the model, we index occupations by $j = 0, \ldots, 9$ and assume the following log-supermodular technology:

$$b(h,j) = \begin{cases} \bar{h} = 1 & \text{for } j = 0, \\ h - \chi_j & \text{for } j \in \{1,\ldots,9\}, \quad 0 = \chi_1 < \chi_2 < \ldots < \chi_9. \end{cases}$$

The characterization of the equilibrium is exactly the same, but we now obtain closed-form solutions. This technology implies that, for the lowest-skill task 0, the worker's skill does not matter and everyone performs the task with equal efficiency. All skills are used in task 1, but for tasks $j \in \{2, \ldots, 9\}$ there is a "skill loss," which increases with higher-order tasks.[26] With 10 discrete tasks, we only need to solve for 10 manager-worker thresholds $\hat{h}_j$, as opposed to Proposition 1 in which we have to solve a differential equation—(40) in the appendix.[27]

Bidimensional skill distribution For the quantitative analysis, we assume a skill distribution that is type IV bivariate Pareto (Arnold, 2014), with the c.d.f.

$$\mu(h,z) = 1 - \left[1 + h^{1/\gamma_h} + z^{1/\gamma_z}\right]^{-a}.$$

We normalize $\gamma_z = 1$, since we cannot separately identify both skills from the skill-specific TFP's. This distribution is consistent with an establishment size distribution that is Pareto, and a wage distribution that is hump-shaped and has a thinner tail. Appendix Figure 23 shows the marginal distributions of h and z.

4.2 Aggregate Production Function

The aggregate production function (4) is estimated outside of the model. For the estimation, we only look at manufacturing (inclusive of mining and construction) and services (inclusive of government). We estimate the parameters γ_m and ϵ from:

$$\begin{aligned}\log\left(\frac{p_m Y_m}{PY}\right) &= \log\gamma_m + (1-\epsilon)\log p_m - \log\left[\gamma_m p_m^{1-\epsilon} + \gamma_s p_s^{1-\epsilon}\right] + u_1 \\ \log(Y) &= c + \frac{\epsilon}{\epsilon - 1}\log\left[\gamma_m^{\frac{1}{\epsilon}} Y_m^{\frac{\epsilon-1}{\epsilon}} + \gamma_s^{\frac{1}{\epsilon}} Y_s^{\frac{\epsilon-1}{\epsilon}}\right] + u_2\end{aligned}$$

where $\gamma_s \equiv 1 - \gamma_m$, using non-linear seemingly unrelated regression on all years of real and nominal sectoral output observed in the BEA Industry Accounts.[28] Real production by sector is computed by a cyclical expansion procedure as in Herrendorf et al. (2013) using production value-added to merge lower-digit industries (as opposed to consumption value-added in their analysis).

Sectoral prices are implied from nominal versus real sectoral quantities, which may depend on the choice of base year. For robustness, we check results using three different base years, corresponding to columns (1)-(3) in Table 1. For each column, respectively, 1947 is the first year the required data is available, 1980 is the first year in our model,

[26] This can be interpreted as low-order skills not being used in high-order tasks, or high-order tasks requiring a fixed cost of preparation to perform the task, resulting in less skills utilized. By assuming task 0 productivity to be constant, we can normalize all other tasks by task 0, as we did for the continuous model.

[27] Characterization of the discrete model is summarized in Appendix D.

[28] The constant c is included since it is not levels but relative changes that identify ϵ.

	(1)	(2)	(3)
γ_m	0.371	0.346	0.258
	(0.003)	(0.005)	(0.004)
ϵ	0.003	0.002	0.003
	(0.000)	(0.000)	(0.004)
AIC	-550.175	-551.264	-550.866
RMSE$_1$	0.106	0.106	0.106
RMSE$_2$	0.039	0.039	0.039

Table 1: Aggregate Production Function

The manufacturing share parameter γ_m and the manufacturing-services elasticity parameter ϵ are estimated from the time series of output and price ratios from 1947 to 2013, from the National Industry Accounts. Standard errors in parentheses. For details of the estimation, we closely follow Herrendorf et al. (2013).

and 2005 is chosen as a year close to the present but before the Great Recession. The values are in a similar range as in Herrendorf et al. (2013). For the calibration, we use the values of (γ_m, ϵ) in column (1).

The capital income share α is computed as the average of 1-(labor income/total income), and fixed at 0.361, and assumed to be equal across sectors.[29] Total income is GDI net of Mixed Income and Value-Added Tax from NIPA and Industry Accounts, and labor income is from NIPA. For the calibration we also need total capital stock (for manufacturing and services) for each decade, which we take from the Fixed Assets Account Table 3 and directly plug into the model.[30] Since we do not model population growth, in practice we normalize output per worker in 1980, y_{1980}, to one, and plug in $K_t = k_t/y_{1980}$ for $t \in \{1980, 1990, 2000, 2010\}$, where k_t is capital per worker in year t.

4.3 Setting Parameters

All parameters are in Table 2, except for the skill loss parameters χ_j, task intensity parameters (η_i, ν_{ij}), and task-TFP growth rates m_j, which are in Table 3. Below, we explain how these parameters are recovered. Appendix D has more detail.

Calibrating the distribution For given γ_h, a, and $\{\chi_j\}_{j=2}^9$, we can numerically compute the thresholds $\{\hat{h}_j\}_{j=1}^9$ and $\hat{z}$ that exactly match observed employment shares by occupation in 1980, by integrating over the skill distribution. With these thresholds, we can compute model-implied relative wages using the discrete version of (25), which is (59) in Appendix D:

$$\frac{w_1 \bar{h}_1}{w_0} = \frac{\bar{h}_1}{\hat{h}_1}, \quad \frac{w_z \bar{z}}{w_0} = \frac{\bar{z}}{\hat{z}}, \quad \text{and} \quad \frac{w_{j+1}(\bar{h}_{j+1} - \chi_{j+1})}{w_j(\bar{h}_j - \chi_j)} = \frac{\bar{h}_{j+1} - \chi_{j+1}}{\bar{h}_j(1 - \chi_{j+1}/\hat{h}_{j+1})}.$$

[29] The difference between the α's when we let them differ between the two sectors was negligible. Herrendorf et al. (2015) compares this assumption against sectoral production functions that are CES in capital and labor, and finds that both specifications capture the effect of differential productivity growth across sectors equally well.

[30] Real capital stock is aggregated using the same cyclical expansion procedure used for value-added.

		Parameter	Value	Target
(A)	Fixed from data	K_{1980}	2.895	Computed from BEA NIPA data
		K_{2010}	4.235	
		α	0.361	
		γ	0.371	Estimated in Section 4.2
		ϵ	0.003	
(B)	Fit to 1980	$M_j \equiv M$	0.985	Output per worker, normalization
		A_m	1.112	Manufacturing employment share
		A_s	1.000	Normalization
		ν_{ij} (18)	Table 3	Witin-sector employment shares by occupation
		η_i (2)		Within-sector manager share
		χ_j (8)	Table 3	Relative wages by occupation
		a	10.087	
		γ_h	0.216	
		γ_z	1.000	Normalizations;
		$\bar{h}$	1.000	Not separately identified from M_j
(C)	Fit to 2010	σ	0.704	Witin-sector employment shares by occupation
		ω	0.341	
		m_j	Table 3	Output per worker growth and within-sector employment shares by occupation

Table 2: Parameters

The population size is normalized, so K_t is capital per capita. All employment share and relative wage targets are from the census, tabulated in Appendix A Table 5.

Here, $\bar{h}_j$ and $\bar{z}$ denote the mean skills in each task. The left-hand side is the ratio of mean wages by occupation, which we observe in the data. The right-hand side is a function only of the thresholds, which themselves are functions of (γ_h, a, χ_j). Hence, we iterate over γ_h, a, and $\{\chi_j\}_{j=2}^{9}$ so that the model-implied ratios match observed mean wage ratios exactly, while at the same time computing the implied thresholds $\{\hat{h}_j\}_{j=1}^{9}$ and $\hat{z}$ that exactly fit 1980 employment shares by occupation.[31]

Similarly, once the skill distribution is fixed, we can compute the implied thresholds that fit 2010 employment shares by occupation. Denote these two sets of thresholds as $\mathbf{x}_{1980}$ and $\mathbf{x}_{2010}$, respectively. These thresholds are determined solely by the exogenously assumed skill distribution and the data, independently of our model equilibria, so they are fixed for the rest of the calibration. We then calibrate the other parameters so that the implied thresholds $\mathbf{x}_{1980}$ and $\mathbf{x}_{2010}$ are consistent with the 1980 and 2010 equilibria, respectively.

Calibrated within the model We have already normalized $(\gamma_z, \bar{h}) = 1$ and $\chi_1 = 0$. We also normalize $A_s = 1$, since the model only implies a relative TFP between sectors, and $M_j \equiv M$ for all $j \in \{0, 1, \ldots, z\}$ for 1980, since they are not

[31] The calibration yields a near linear increase in the skill loss parameters χ_j with j.

Ranked by mean wage	χ_j	Emp Wgts (ν_{ij}, η_i) Manu.	Serv.	m_j (%)
Low Skill Services	**-**	**0.016**	**0.136**	**-0.731**
Middle Skill		**0.816**	**0.524**	
Administrative Support	-	0.088	0.173	2.930
Machine Operators	0.001	0.256	0.015	9.122
Transportation	0.002	0.119	0.081	4.348
Sales	0.003	0.026	0.123	0.012
Technicians	0.005	0.034	0.040	-1.144
Mechanics & Construction	0.006	0.159	0.065	2.315
Miners & Precision Workers	0.007	0.134	0.027	6.328
High Skill		**0.168**	**0.340**	
Professionals	0.009	0.070	0.195	-2.248
Management Support	0.010	0.098	0.146	-0.489
Management	**-**	**0.076**	**0.130**	**-0.017**

Table 3: Calibrated Employment Weights and Growth Rates

separately identified from (η_i, ν_{ij}) in a static equilibrium. This follows from the production technology we assume in (7)-(8). We denote the 1980 levels of the TFP's by (M, A_i) and their 2010 levels by multiplying them by their respective growth rates. For example, the manager-task TFP in 2010 is $M(1+m_z)^{30}$ and similarly sector i's TFP in 2010 is $A_i(1+a_i)^{30}$.

This leaves us with 35 parameters to be calibrated: the elasticity parameters (σ, ω), TFP parameters (M, A_m), task intensities η_i and $\{\nu_{ij}\}_{j=1}^{9}$ for $i \in \{m, s\}$, and the task-TFP growth rates $\{m_j\}_{j=z,0}^{9}$.[32] Since we can solve for the discrete version of the model equilibrium in closed form, most parameters are chosen so that our 1980 equilibrium exactly fits the 1980 data moments in Appendix Table 5.

Then except for capital per worker, which we plug in from the data, all other parameters are held fixed and only M_j, the exogenous task-TFP's, grow from 1980 to 2010 at rate m_j. In particular, our benchmark scenario assumes that $a_m = a_s = 0$.

The 11 constant task-TFP growth rates $\{m_j\}_{j=z,0}^{9}$ and elasticity parameters (σ, ω) are chosen to fit the time trends of aggregate output per worker growth and employment shares *within sectors* from 1980 to 2010 (13 parameter, 21 moments).[33] All resulting parameters are tabulated in Tables 2 and 3.

Discussion As implied by the data in Figure 20(a) and Appendix Table 5, the manufacturing sector has higher intensity parameters among middle skill jobs and a

[32]There are only 9 horizontal intensity parameters to calibrate per sector, since $\sum_{j=0}^{9} \nu_{ij} = 1$.

[33]We target the linear trend from 1980 to 2010 rather than their exact values. However, since most trends are in fact linear, using the exact values barely change our results.

lower intensity in managers.[34] Since the estimated elasticity between manufacturing and services is less than one, for structural change to occur, productivity needs to rise by relatively more in those occupations used more intensively by manufacturing, which are the middle-skill jobs. This is evident from the last column in Table 3.

The calibrated values for $\omega = 0.34 < \sigma = 0.70 < 1$ are important both for Section 3 and our quantitative results to follow. While this was a sufficient condition in Section 3, it is validated by the data. The only other paper we know of that recovers the elasticity across tasks is Goos et al. (2014). Their point estimate for σ is around 0.9, which is much closer to 1 than ours.[35] However, theirs is an empirical framework that does not take into account general equilibrium or aggregate effects. Both in their model and ours, the employment share change of occupation j is determined by $(1-\sigma)m_j$ (Appendix C.4). If we were to set $\sigma = \omega = 0.9$, we would recover much higher values for m_j to explain the employment share changes in the data. This would result in sectoral and aggregate TFP growth rates that are unrealistically high.

5 Quantitative Analysis

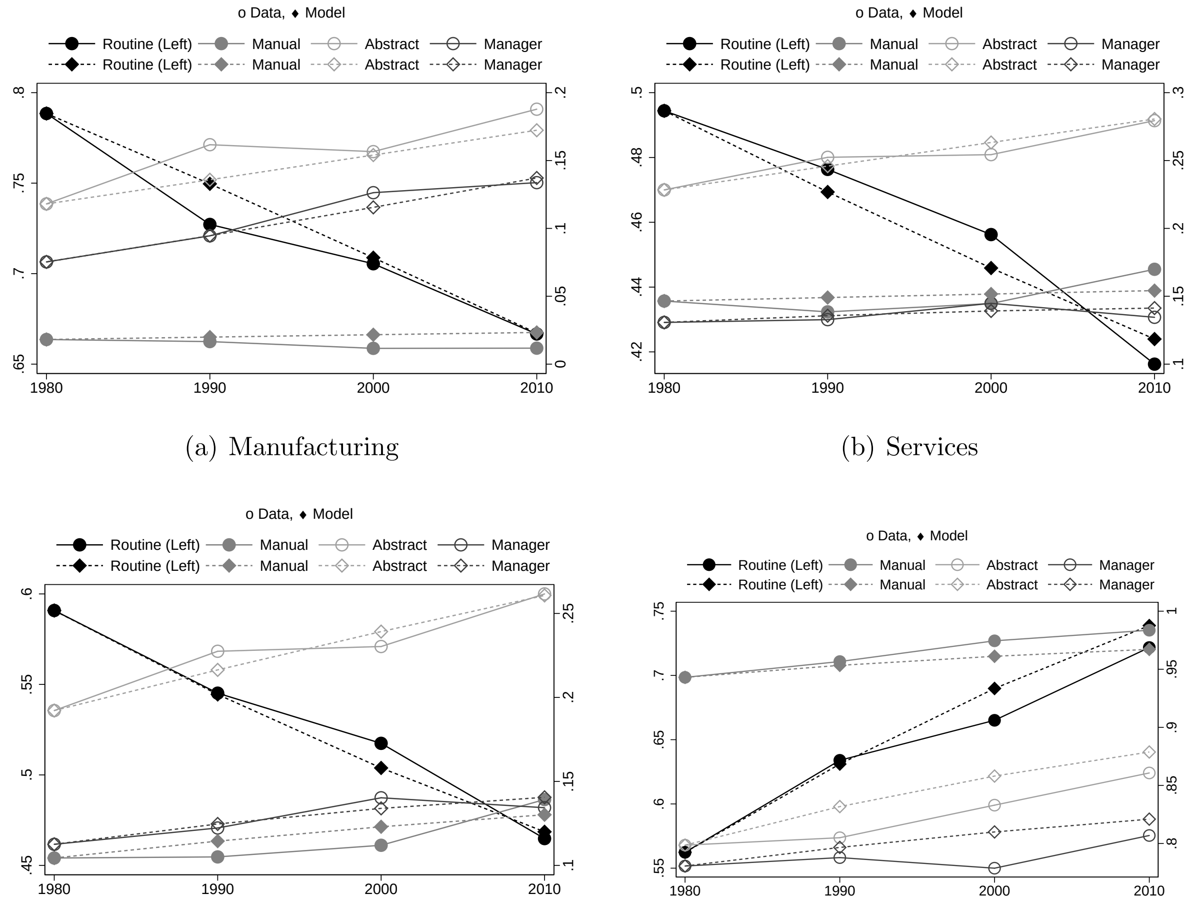
We first assess how well the model fits empirical trends in within-sector occupation employment shares, which were targeted, and then its performance in untargeted dimensions. Then, to contrast TSTC against sector-specific technological changes, we compare the benchmark model against versions that also allow exogenous growth in sectoral TFP's. Finally, we relate the rate of TSTC quantified from our model to empirical measures of occupational characteristics.

5.1 Model Fit

Figure 10 plots the model implied trends in employment shares across tasks, in aggregate and by sector, against the data. When computing the simulated paths for 1990 and 2000, we plug in the empirical values of $K_t = k_t/y_{1980}$ and the task-specific TFP's implied by the calibrated growth rates, and solve for the respective equilibrium allocations. For ease of graphical representation, the figure groups the 10 worker occupations ranked by their 1980 mean wage into 3 broader categories summarized in Table 3: manual, routine, and abstract.

At first glance, it may not be so surprising that we obtain a more or less exact fit as seen in Figure 10(c)-(d), since the discrete model equilibrium can be solved in closed form for any given year, as we explain in Appendix D. However, while we target

[34] Since we normalize $M_j \equiv M$ in 1980, the parameters correspond to skill-adjusted employment weights.
[35] Since manager is not a special occupation in that paper, it lacks a counter part to our ω.



(a) Manufacturing

(b) Services

(c) Aggregate

(d) Services Share By Task

Fig. 10: Data vs. Model, Employment Shares by Task.
The quantitative model has 11 one-digit occupation groups. For graphical representation only, we re-group the 10 worker occupations into the 3 broader categories of manual, routine, and abstract as in Table 3. The vertical axes in (a)-(c) are employment shares for each occupation group, routine on the left and the rest on the right. The vertical axes in (d) are the fraction working in services for each occupation group.

the starting points for all the shares (services employment share, and within-sector employment shares by task), we calibrated 21 trends using only 13 parameters: the 2 elasticity parameters (σ, ω) and 11 task-specific (and sector-neutral) growth rates, as shown in Panel (C) of Table 2. The quantitative model predicts that both horizontal and vertical polarization are faster in the manufacturing sector, as we saw in the data and as implied by our theory: The routine employment share falls by 13 percentage points in manufacturing vs. 8 in services, while the manager employment share rises by 6 percentage points in manufacturing vs. 1 in services.

Furthermore, we did not target any aggregate or sectoral employment shares, so the fact that aggregate occupation shares and structural change by occupation (i.e., the rise in service share for each occupation group) are almost exactly replicated, as

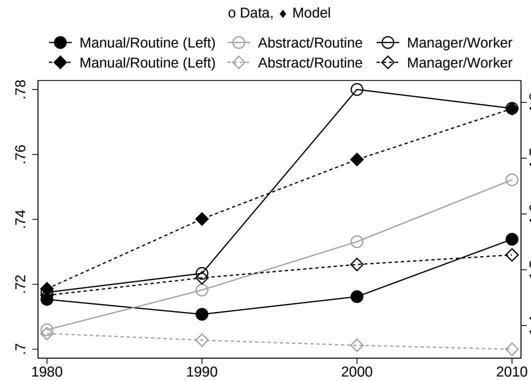


Fig. 11: Data vs. Model, Relative Wages by Task.
The quantitative model has 11 one-digit occupation groups. For graphical representation only, we re-group the 10 worker occupations into the 3 broader categories of manual, routine, and abstract as in Table 3. The vertical axes are the ratios between the average wages of occupation groups, manual-to-routine on the left and all the others on the right.

seen in Figure 10(c)-(d), is also a success of the quantitative model. This suggests that the consequences of TSTC implied by Lemmas 2-4 should be sufficient for explaining structural change.

5.2 Implications for Moments Not Targeted

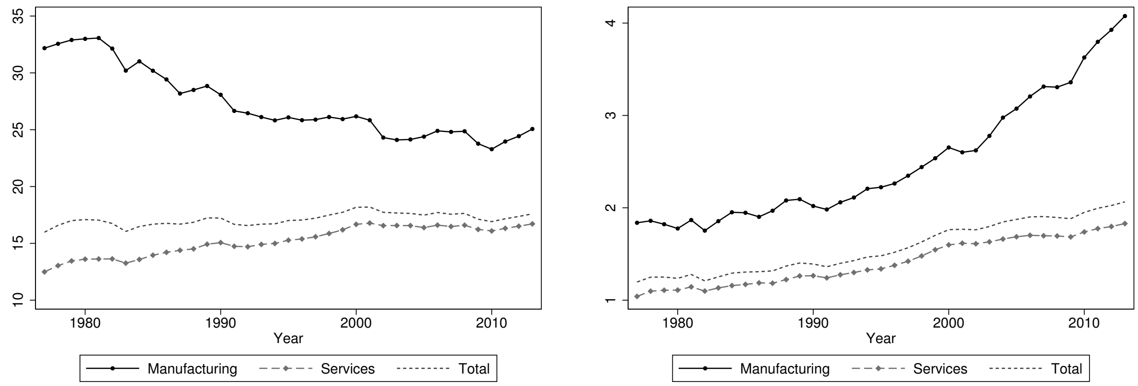
Relative wages Although we target the 1980 average wages of the 11 occupation groups in our calibration, we do not exploit any other wage-related moments. Before we further discuss the model's implications for wage moments, two clarifications are in order. First, in our model, individuals' earnings depends only on their skills and occupation, not on sectors. In the data, the average wage even for a narrowly-defined occupation is somewhat higher in manufacturing than in services—for example, see Appendix Figure 22(b).[36] We do not directly address this fact, and the average wages of broad occupation groups (e.g., workers as a whole or the manual, routine, abstract categories) are different between the two sectors only to the extent that they differ in how intensely they use the 10 underlying worker occupations in Table 3.[37]

Second, Proposition 2 leads to polarization cast in terms of wage per skill (w_z and w_h). Our unit of observation is now wages, which equal (wage per skill)$\times$(amount of skill). Accordingly, the average wage of an occupation changes not only because of changes in its per-skill wage, but also because of selection on skill by occupation.

Figure 11 plots the relative mean wages of occupations (in aggregate). Manual

[36]However, the sectoral difference in the average wage of an occupation is stable throughout the observation period, so we consider the indifference assumption to be valid up to a constant.

[37]In this version of the model with discretized worker occupations, we only consider those equilibria in which mean skill levels within occupations are equal across sectors.



(a) Average No. of Workers (b) Average Value-added Output

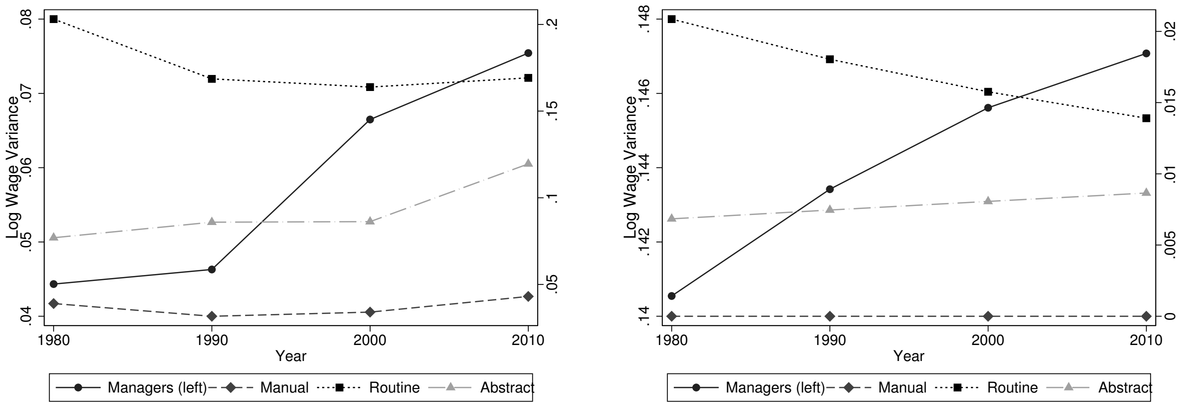
Fig. 12: Establishment Size by Sector

Source: BDS and NIPA. Left: Average number of workers per establishment, in aggregate and by sector. Right: Real value-added output by establishment (millions of 2010 USD), in aggregate and by sector.

and abstract wages are relative to routine jobs, and manager wages are relative to all workers. While the model trends are qualitatively consistent with the data, i.e., horizontal and vertical wage polarization, the quantitative fit is not tight. In particular, the rise in the relative wage of the manager and abstract occupations is more muted than in the data, which is explained by *negative* selection. As shown by Proposition 2 and Figure 8, barring a change in the underlying distributions of skills, vertical polarization dictates that new managers have less managerial skill z than existing ones. This brings down the mean skill level of managers, countering the positive impact on the average wage of managers coming from the rise in wage per managerial skill w_z. Likewise, horizontal polarization necessarily implies lower-h workers in the highest-paying abstract jobs, attenuating the positive impact on their average wage from the higher wage per skill (w_j for $j = 8, 9$).[38]

Average size of establishments In our model, the production technology has constant returns to scale, and the size distribution of establishments is not pinned down. However, because we model managers as a special occupation qualitatively separate from workers, the model does have implications on the average size of establishments as long as we assume a stable relationship between managers and establishments. With such an assumption, e.g., a constant number of managers per establishment over time, the faster vertical polarization in manufacturing, Figure 5(b), implies that the number of workers per establishment should fall faster in manufacturing than in services. This is

[38]Such selection does not matter for manual jobs ($j = 0$), because our discretized model assumes that all workers contribute $\bar{h} = 1$ toward task-0 production regardless of their h. The increase in the manual job wage is entirely due to the higher wage per effective skill, w_0.



(a) Data

(b) Model

Fig. 13: Within-Occupation Wage Inequality

Log wage variance within the 4 occupation groups in the data and the model. Left scale for managers and right scale for the rest.

confirmed in Figure 12(a), which plots the average number of workers per establishment in the Business Dynamics Statistics (BDS) from the U.S. Census Bureau.

Furthermore, since the model generates vertical polarization by faster productivity growth of routine jobs, which are more intensively used in manufacturing, the faster vertical polarization in manufacturing is accompanied by higher productivity and output growth among manufacturing establishments. In Figure 12(b), the sectoral value-added in the NIPA is divided by the number of establishments in the BDS, which confirms the model prediction.

An inconsistency is that, while the employment share of managers has grown in the data, the number of employees per establishment overall has stayed more or less constant throughout the observation period. This suggests a need for modeling differentiated managerial occupations and hierarchies of management.

Within-group wage inequality Because our quantitative model has a continuum of skills and discrete occupations, it has implications for wage inequality *within* occupation groups as well. As shown in Figure 13(a), log wage variances rose substantially among managers, slightly among abstract workers, dropped among routine workers and remained more or less constant among manual workers.

These qualitative patterns are replicated in our model, but the magnitudes of the changes are too small compared to the data. One way to address this is to increase the variance of the underlying skill distribution over time, which we decided against in order to isolate task specific forces: We would be unable to separate the change in the skill distribution that is task-specific as opposed to task-neutral, without further assumptions on how skill is accumulated.

5.3 Polarization, Structural Change and TFP's

The preceding subsections show that the model targeted only to within-sector employment shares delivers a good fit in terms of employment shares in the overall economy, albeit less in terms of relative wages. Other non-targeted moments such as establishment size and wage inequality within occupations are also qualitatively consistent with the data. We now focus on sectoral employment shares and TFP's.

To be more specific, we analyze the role of TSTC on structural change in relation to two counterfactuals.

(1) We restrict all task-specific TFP growth to be equal, $m_j = m$ for all j, and instead let the exogenous sector-specific TFPs (A_m, A_s) grow at rates a_m and a_s, respectively. We jointly recalibrate m, a_m, and a_s to match the empirical growth rate of the aggregate and sectoral TFP's from 1980 to 2010. This version only has exogenous sector-specific TFP growth but no TSTC.[39]

(2) We allow both exogenous task- and sector-specific TFP growth, and recalibrate $\{m_j\}_{j=z,0}^{9}$, a_m, and a_s to match the change in employment shares *and* the empirical growth rates of the aggregate and sectoral TFP's from 1980 to 2010. Recall that our benchmark calibration of Section 4 restricted a_m and a_s to be 0 but did not target sectoral TFPs.

In both cases, we keep all other parameters at their benchmark values of Tables 2-3, and only recalibrate the TFP growth rates.

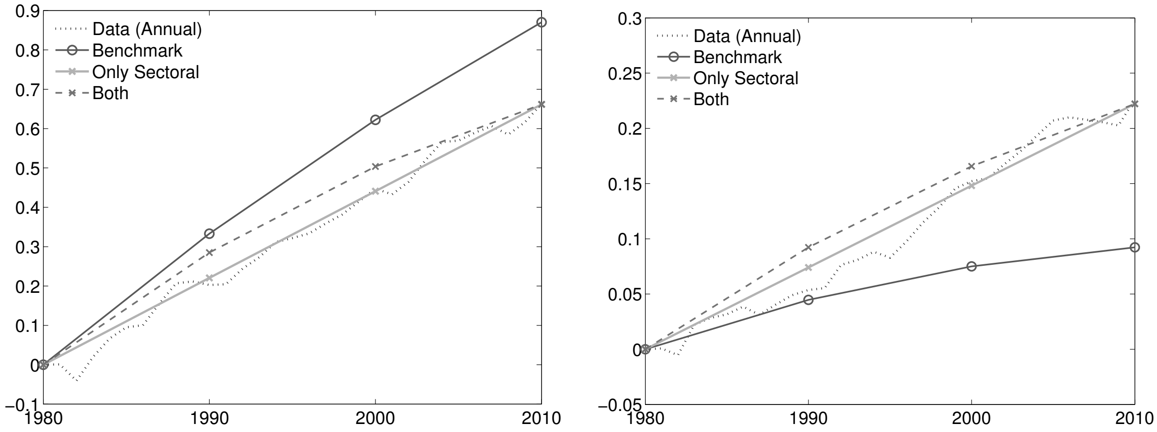
We focus on sectoral TFP's since in our model structural change results from the differential TFP growth between sectors—expressed in closed form in (20)—whether it is exogenous (caused by a_m and a_s) or endogenous (as in Section 3.2). The recalibrated parameters for the counterfactual exercises are in Appendix Table 6.

TFP and output growth Figure 14 shows the paths of sectoral log TFP's in the data, in our benchmark calibration, and in the two counterfactuals. By construction, all four match aggregate TFP and GDP growth over time (Appendix Figure 24).[40]

Instead, we focus on the evolution of sectoral TFP's. Note that the calibrated TSTC rates are higher among routine jobs, on which manufacturing is more reliant.

[39] Sectoral TFP is constructed from the NIPA accounts. Real value-added and capital are computed via cyclical expansion from the industry accounts, labor is computed from full-time equivalent persons in production in NIPA Table 6, and TFP is the Solow residual by sector.

[40] Denoting aggregate TFP as Z_t, since $Y_t = Z_t K_t^{\alpha}$ (labor is normalized to one) and we plug in the empirical values of K_t for all calibrations, it is the same whether we match aggregate TFP or GDP.



(a) Log TFP, Manufacturing

(b) Log TFP, Services

Fig. 14: Benchmark vs. Counterfactuals, Sectoral TFP.
Data: NIPA. Log 1980 levels are normalized to 0, so the slopes of the lines are the growth rates.

Since the elasticity between sectors (ϵ) is less than 1, we know from Lemma 3 that the higher endogenous TFP growth in manufacturing leads to structural change. However, our benchmark calibration did not target sectoral TFP's, so the question is whether their growth can be explained by our benchmark with only TSTC.

In our benchmark, we overshoot the growth rate of manufacturing TFP by about half a percentage point per annum, while undershooting the services TFP growth rate by the same magnitude. However, when we look at the growth rates of sectoral output (Figure 25), these gaps nearly disappear. This is because while the model assumes that capital and labor input ratios are equal across sectors, as shown in (22), in the data they are not. In fact, counterfactuals (1) and (2) in Appendix Figure 25 show that when sectoral TFP growth is matched exactly, manufacturing output grows more slowly, and services output more quickly, than in the data. This implies that the capital input ratio between manufacturing and services grew faster than the labor input ratio, although the differences are small. Consequently, once we include exogenous sector-specific TFP growth and target the empirical sectoral TFP's, both counterfactuals under-predict manufacturing output and over-predict services output.

Structural change and polarization Since structural change from manufacturing to services is solely determined by sectoral TFP ratios (Lemma 3), the fact that endogenous sectoral TFP growth in our benchmark closely tracks the data implies that our model will also explain structural change in terms of employment shares. As shown in Figure 15, our benchmark overshoots the 13-percentage-point rise in the service employment share in the data by 1 percentage point, while both counterfactuals (1) and (2) undershoot by 3 percentage points. Moreover, Appendix Figure 26 shows

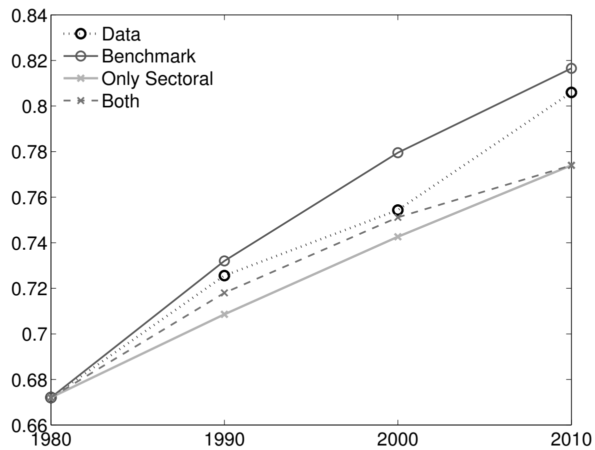


Fig. 15: Benchmark vs. Counterfactuals, Service Employment Share.
Vertical axis is the services share of overall employment.

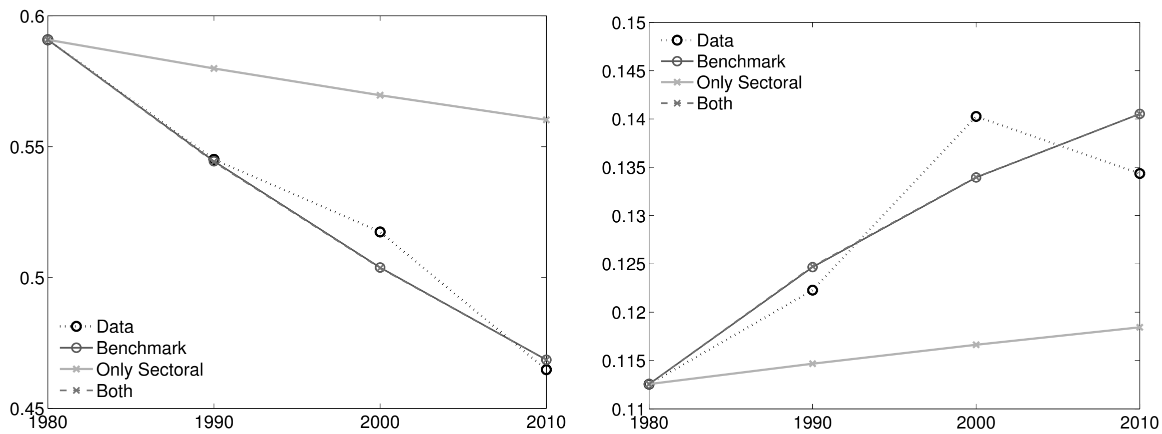
that when we look at structural change within occupation groups, the benchmark outperforms both counterfactuals (1) and (2), especially for managers.

The benchmark better fits employment shares than the counterfactuals, despite overshooting manufacturing's relative TFP growth, because it has a better fit to sectoral output growth. As explained above, in the counterfactuals, sectoral output growth is too low in manufacturing and too high in services. To the extent that all structural change in our model is due to differential growth in sectoral TFP's, we do not intend to emphasize too much that the benchmark explains employment shares better than the counterfactuals that explicitly target sectoral TFP's.

We emphasize that exogenous changes in sectoral TFP's cannot cause *within*-sector polarization. We still investigate their effect on aggregate employment shares by occupation (Lemma 4). Figure 16 shows that sectoral forces alone can account for 15-20 percent of horizontal and vertical polarization in aggregate. This is slightly smaller than the simple calculation in Section 3.3, which showed that the effect of sector-specific TFP's on polarization is modest because both sectors use routine jobs.

In contrast, TSTC (m_j's) can account for almost all of the changes in both occupational *and* sectoral employment shares. At first glance, it may seem that the effect of TSTC on sectoral TFP's—which drive structural change—should also be modest since services also benefits from the faster TFP growth of routine tasks. However, TSTC is accompanied by a reallocation of heterogeneous individuals across occupations, which endogenously reinforces the exogenous shifts.[41]

[41] In Lemma 3, this is shown as the faster growth rates of the endogenous TFP components (Π_{K_i}, Π_{L_i}) in *services*, which implies faster TFP growth in *manufacturing*, given the expression for TFP (20) and assumed values of elasticities. Details are in Appendix C.5.



(a) Routine Employment Share (b) Manager Employment Share

Fig. 16: Benchmark vs. Counterfactuals, Polarization

Vertical axes are the routine occupation share of total employment (left) and the manager share of total employment (right). The declining routine share represents horizontal polarization and the rising manger share vertical polarization.

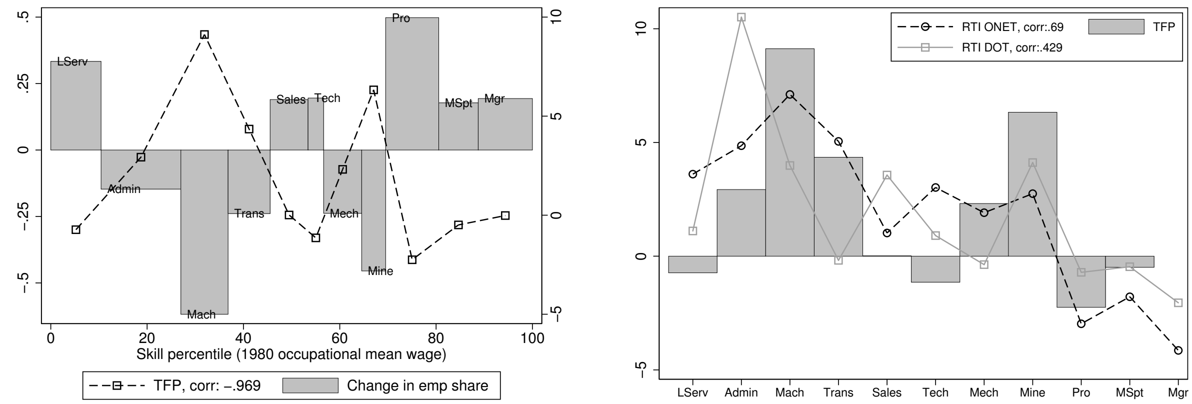
To summarize, TSTC almost fully accounts for sectoral TFP growth and hence structural change observed between 1980 and 2010. Due to the vertical and horizontal polarization induced by TSTC, employment shifts to the sector that relies less on routine tasks and more on management. Conversely, sector-specific productivities can only account for 15-20 percent of polarization in the overall economy; more important, we have shown that they cannot cause polarization within sectors, contrary to the data.

5.4 What Explains Task-Specific Productivity Growth?

Even with skill selection, horizontally and vertically differentiated occupations, and multiple sectors, Figure 17(a) shows that the bulk of the changes in occupational employment shares are directly accounted for by task-specific TFP's, with a correlation coefficient of -0.97. This is also confirmed by the regression in the top panel of Appendix Table 7. This leads us to conclude that in order to understand changes in the employment structure, it is important to identify what these task-TFP's represent.

How much of the variation in the rates of TSTC can be explained by the widely-accepted routinization hypothesis—that routine jobs were more easily automated and hence now employ fewer workers? As a first pass, in Figure 17(b) we correlate the task-TFP growth rates with the RTI index used in Autor and Dorn (2013), which aggregates indices used in Autor, Levy, and Murnane (2003), which in turn were constructed by aggregating over task requirements for specific jobs in the DOT.[42] We also correlate

[42]Figure 20(b), replicated from Autor and Dorn (2013), shows where the top employment-weighted third of occupations in terms of RTI are along the skill percentiles. Because most routine jobs are found in the middle, it is hypothesized and then formally tested that routinization causes (horizontal) job polarization.



(a) Task-TFP Growth and Employment Shares

(b) Task-TFP Growth and RTI

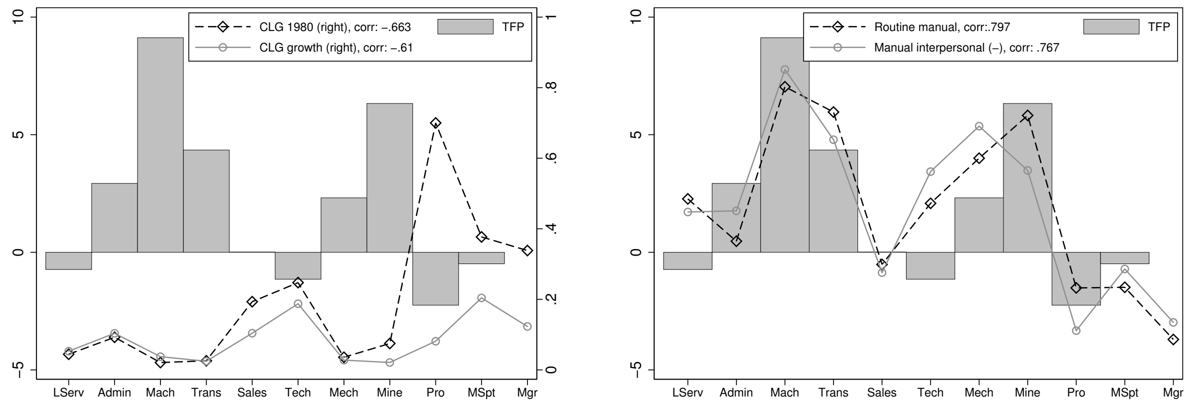
Fig. 17: Employment Shares, Task TFP Growth and Routinization Index. Panel (a): x-axis is percent share of 1980 aggregate employment. For employment share changes, the left vertical axis is the percentage change per percentile. For task-TFP growth rates, the right vertical axis is percent per year. Panel (b): RTI indices; and task-TFP growth rates in percent per year.

them with the RTI index from Acemoglu and Autor (2011), which was constructed similarly but instead using O*NET, the successor to DOT.

While the task-TFP growth rates are positively correlated with both RTI indices across occupations, and more strongly with the latter, there is much left to be explained. Both the correlation and R^2's are still quite low, as shown in Appendix Table 7.

What about variables related to college education? The skill-biased technological change (SBTC) literature proxies skill by a (four-year) college degree—see Acemoglu (2002) for a review. As is evident from Figure 18(a), neither the fraction of college graduates within each occupation in 1980, nor the change in this fraction from 1980 to 2010, has much of a relationship with the task-specific TFP growth rates. Although not shown here, the level in 1980 and the growth between 1980 and 2010 of within-occupation college wage premium are not correlated with the task-specific TFP growth either. Moreover, as shown in Appendix Table 7, the correlation between task-specific productivity growth and college-related variables is *negative* across occupations; that is, those occupations with more college graduates or those in which the college graduate share grew the fastest in fact became relatively less productive. We conclude that college-related variables do not explain the employment shifts across occupations and sectors between 1980 and 2010. This is in contrast to the preceding period: Katz and Murphy (1992) finds that college variables can account for the changes in occupational employment shares from the early 1960s to mid-1980s, which our own empirical analysis affirms (not reported here).

What we do find, however, is that the task-specific TFP growth rates correlate



(a) Task-specific TFP Growth and College

(b) Task-specific TFP Growth and O*NET

Fig. 18: Task TFP Growth, College Shares and O*NET-based Indices.
In both Panel (a) and (b), gray bars are the task-TFP growth rates in percent per year. Panel (a) has within-occupation fraction of college graduates in 1980 and changes in this fraction across occupations. Panel (b) shows occupation-level routine-manual and manual-interpersonal indices in O*NET.

strongly with sub-indices constructed by Acemoglu and Autor (2011) using O*NET, rather than the RTI index which aggregates over them. As shown in Figure 18(b), the correlation of the task-TFP growth rates with routine-manual and manual-interpersonal indices of occupations, the latter of which is not in RTI, is 0.80 and -0.77 respectively. Appendix Table 7 shows that the R^2 of the regressions is high at 0.64 and 0.59.

We conclude that the technological progress since 1980 has predominantly enhanced the productivity of those occupations that are heavy on routine-manual tasks but light on interpersonal skills, shrinking their employment shares and relative wages. Since routine physical activities are easy to automate but tasks requiring interpersonal skills are not, this finding points to automation as the channel of TSTC.[43] In this context, our findings are consistent with the routinization hypothesis. The unexplained part of task-specific TFP growth may also come from endogenous changes in the distribution of manager and worker skills, heterogeneous degrees of capital-labor substitutability across tasks,[44] and offshoring for an open economy, all of which we abstract from.[45]

[43]As discussed at the beginning of Section 3, the effect of a change in factor-neutral task-TFP is qualitatively similar to that of a capital-augmenting change, which for example could have been modeled as the fall in the price of task-specific capital (Goos et al., 2014) and directly interpreted as automation.

[44]That is, the elasticity of substitution between capital and workers' human capital may vary across tasks. This would be related but distinct from typical models of capital-skill complementarity in which the elasticity varies directly by skill, e.g. low- vs. high-skill as in Krusell, Ohanian, Ríos-Rull, and Violante (2000).

[45]In an open economy setting, cheaper foreign labor would be qualitatively similar to higher productivity at the task level. We note that, although not shown here, the task-TFP growth rates are only weakly correlated with occupations' offshorability index constructed by Firpo, Fortin, and Lemieux (2011).

6 Conclusion

We constructed a tractable yet powerful framework for studying the occupational, industrial and organizational structure of an economy. Theoretically, we fully characterize the equilibrium and prove that TSTC for middle-skill jobs leads to horizontal polarization, vertical polarization, and structural change. Empirically, we document that polarization is prevalent within both the manufacturing and service sectors, and faster in the former, whose relative TFP growth is also higher. Quantitatively, we show that TSTC alone can fully account for all of the above phenomena in the data, unaided by sector- or factor-specific technological change. We also show that the occupations with the highest rate of TSTC are more routine-manual and less interpersonal in nature.

Our model is suitable for many potential extensions. One could embed individual skill dynamics to separate task productivity growth from human capital accumulation, or include differentiated managerial tasks that match with different sets of worker tasks—this would also be useful for studying within- and between-firm inequality. A quantitative analysis with more than two sectors would facilitate a sharper decomposition of occupation- and industry-specific changes, as would an analysis at a higher frequency (e.g., annual rather than decadal). A multi-country extension is also feasible, which could be used to analyze trade, offshoring, and foreign direct investment. We are actively exploring some of these exciting topics.

References

Acemoglu, D. (2002): "Technical Change, Inequality and the Labor Market," *Journal of Economic Literature*, 40, 7–72.

Acemoglu, D. and D. Autor (2011): "Skills, Tasks and Technologies: Implications for Employment and Earnings," in *Handbook of Labor Economics*, ed. by D. Card and O. Ashenfelter, Elsevier, vol. 4, Part B, 1043 – 1171.

Acemoglu, D. and V. Guerrieri (2008): "Capital Deepening and Nonbalanced Economic Growth," *Journal of Political Economy*, 116, 467–498.

Arnold, B. C. (2014): "Univariate and Multivariate Pareto Models," *Journal of Statistical Distributions and Applications*, 1, 1–16.

Autor, D. H. and D. Dorn (2013): "The Growth of Low-Skill Service Jobs and the Polarization of the US Labor Market," *American Economic Review*, 103, 1553–97.

Autor, D. H., L. F. Katz, and M. S. Kearney (2006): "The Polarization of the U.S. Labor Market," *American Economic Review*, 96, 189–194.

Autor, D. H., F. Levy, and R. J. Murnane (2003): "The Skill Content of Recent Technological Change: An Empirical Exploration," *The Quarterly Journal of Economics*, 118, 1279–1333.

Bárány, Z. L. and C. Siegel (2018): "Job Polarization and Structural Change," *American Economic Journal: Macroeconomics*, 10, 57–89.

Buera, F. J. and J. P. Kaboski (2012): "The Rise of the Service Economy," *American Economic Review*, 102, 2540–69.

Buera, F. J., J. P. Kaboski, and R. Rogerson (2015): "Skill Biased Structural Change," Working Paper 21165, National Bureau of Economic Research, Inc.

Caliendo, L., F. Monte, and E. Rossi-Hansberg (2015): "The Anatomy of French Production Hierarchies," *Journal of Political Economy*, 123, 809 – 852.

Costinot, A. and J. Vogel (2010): "Matching and Inequality in the World Economy," *Journal of Political Economy*, 118, 747–786.

Dorn, D. (2009): "Essays on Inequality, Spatial Interaction, and the Demand for Skills," Ph.D. thesis, University of St. Gallen.

Dürnecker, G. and B. Herrendorf (2017): "Structural Transformation of Occupation Employment," Manuscript.

Firpo, S., N. M. Fortin, and T. Lemieux (2011): "Occupational Tasks and Changes in the Wage Structure," Discussion Paper 5542, Institute for the Study of Labor.

Gabaix, X. and A. Landier (2008): "Why Has CEO Pay Increased So Much?" *Quarterly Journal of Economics*, 49–100.

Garicano, L. and E. Rossi-Hansberg (2006): "Organization and Inequality in a Knowledge Economy," *The Quarterly Journal of Economics*, 121, 1383–1435.

Goos, M., A. Manning, and A. Salomons (2014): "Explaining Job Polarization: Routine-Biased Technological Change and Offshoring," *American Economic Review*, 104, 2509–26.

Herrendorf, B., C. Herrington, and A. Valentinyi (2015): "Sectoral Technology and Structural Transformation," *American Economic Journal: Macroeconomics*, 7.

Herrendorf, B., R. Rogerson, and A. Valentinyi (2013): "Two Perspectives on Preferences and Structural Transformation," *American Economic Review*, 103, 2752–89.

——— (2014): "Growth and Structural Transformation," in *Handbook of Economic Growth*, ed. by P. Aghion and S. N. Durlauf, Elsevier, vol. 2, chap. 6, 855–941.

Katz, L. F. and K. M. Murphy (1992): "Changes in Relative Wages, 1963-1987: Supply and Demand Factors," *Quarterly Journal of Economics*, 107, 35–78.

Krusell, P., L. E. Ohanian, J.-V. Ríos-Rull, and G. L. Violante (2000): "Capital-skill Complementarity and Inequality: a Macroeconomic Analysis," *Econometrica*, 68, 1029–1054.

Lee, S. Y. T., Y. Shin, and D. Lee (2015): "The Option Value of Human Capital: Higher Education and Wage Inequality," Working Paper 21724, National Bureau of Economic Research, Inc.

Lucas, R. E. (1978): "On the Size Distribution of Business Firms," *Bell Journal of Economics*, 9, 508–523.

Meyer, P. B. and A. M. Osborne (2005): "Proposed Category System for 1960-2000 Census Occupations," Working Paper 383, U.S. Bureau of Labor Statistics.

Ngai, L. R. and C. A. Pissarides (2007): "Structural Change in a Multisector Model of Growth," *American Economic Review*, 97, 429–443.

Stokey, N. L. (2016): "Technology, Skill and the Wage Structure," Working Paper 22176, National Bureau of Economic Research, Inc.

Tervio, M. (2008): "The Difference That CEOs Make: An Assignment Model Approach," *American Economic Review*, 98, 642–68.

Online Appendix

A Census Employment/Wages/Occupations

Occupation Group		`occ1990dd`
Managers	self-employment+	4– 19
Management Support		22– 37
Professionals		43–199
Technicians		203–235
Sales		243–283
Administrative Support		303–389
Low Skill Services		405–472
Mechanics and Construction		503–599
Miners and Precision		614–699
Machine Operators		703–799
Transportation Workers		803–899

Table 4: Census Occupation Groups

322 non-farm occupations according `occ1990dd` (Dorn, 2009), itself harmonized from `occ1990` (Meyer and Osborne, 2005), are grouped into 11 occupation groups in order of their `occ1990dd` code. All other occupation groups correspond to their 1-digit census occupation group except for management support, technicians and sales. Groups are presented in their (contiguous), ascending order of their codes, excluding agricultural occupations 473–498 which are dropped. In the main text, occupation groups are presented in ascending order of skill (mean hourly wage).

We use the 5% census samples from IPUMS USA. We drop military, unpaid family workers, and individuals who were in correctional or mental facilities. We also drop workers who work either in an agricultural occupation or industry.

For each individual, (annual) employment is defined as the product of weeks worked times usual weekly hours, weighted by census sampling weights. Missing usual weekly hours are imputed by hours worked last week when possible. Missing observations are imputed from workers in the same year-occupation-education cell with 322 occupations$\times$6 hierarchical education categories: less than high school, some high school, high school, some college, college, and more than college.

Hourly wages are computed as annual labor income divided by annual employment at the individual level. Hence while employment shares include the self-employed, hourly wages do not include self-employment income.[46] We correct for top-coded incomes by multiplying them by 1.5, and hourly wages are set to not exceed this value divided by 50 weeks $\times$ 35 hours (full-time, full-year work). Low incomes are bottom-coded to first percentile of each year's wage distribution.

For the line graphs in Figures 3–4, we ranked occupations by their hourly wages defined as above, and smoothed across skill percentiles using a bandwidth of 0.75 for employment and 0.4 for wages; these are the same values used in Autor and Dorn

[46]While we have only considered labor income in the paper, we have conducted robustness checks by including business income as well. Hourly business income is defined similarly as hourly wages. We also separately corrected for top-coding (the top-codes for labor and business income differ) and bottom-coding in a similar fashion.

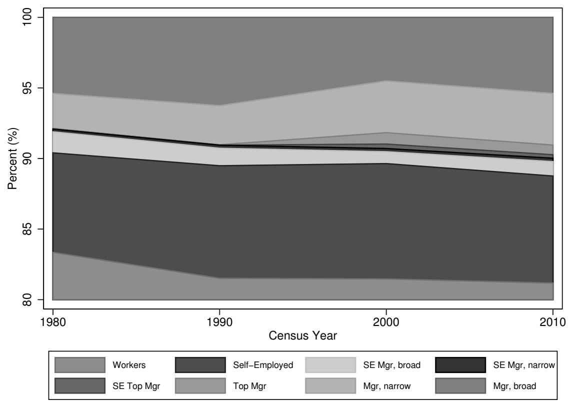


Fig. 19: Managers in the Census

Source: U.S. Census (5%). Top managers are coded 4 in `occ1990dd` while broad managers include code 22 which are not-elsewhere-classified managers, or manager occupations that do not exist across all 4 censuses.

(2013). For the bar graphs in Figures 3–4, 17–18 and 20, we grouped the 322 occupations vaguely up to their 1-digit Census Occupation Codes, resulting with the 11 categories summarized in Table 4 and used for our quantitative analysis. In the figures and in Tables 5–6, these groups are then ranked by the mean wage of the entire group. In particular, in Figures 3–4, 17(a) and 20, the horizontal length of a bar is set to equal the corresponding group's 1980 employment share, which does not necessarily coincide with the 3-digit occupations used to generate the smooth graphs by percentile.

Throughout the paper, we subsume all self-employed workers into the manager group. However, excluding them does not affect any of our quantitative results because the employment share of non-managerial self-employed workers was more or less constant throughout the observation period, as shown in Figure 19. There, we decompose managers into 9 subgroups. Our benchmark definition includes all 3 self-employed groups, top managers and narrow managers, but excludes broad managers. Top managers are coded 4 according to `occ1990dd` and includes CEO's, public administrators and legislators. Broad managers are coded 22 and are either not-elsewhere-classified or manager occupations that do not exist across all 4 censuses.

B Definition of Competitive Equilibrium

All markets are perfectly competitive. Individuals only care about which task they perform and are indifferent across sectors, and choose the task that yields the highest wage. Let $w_h(h)$ denote the wage of a worker with human capital h, and let w_z denote the efficiency wage per unit of managerial skill z for managers—the latter is possible since all managers perform the same task and we assume a constant returns to scale technology. Then an individual with skill $s = (h, z)$ becomes a manager if and only if

$$w_z z \geq w_h(h). \tag{33}$$

The final good is produced by a representative firm that takes sectoral prices as given, whose profits are given by

$$PY - p_m Y_m - p_s Y_s,$$

where Y is given in (4), P is the final good price and p_i the price of the sector $i \in \{m, s\}$ good. Normalizing $P = 1$, profit maximization yields (24) in the main text.

A manager with skill z in sector i maximizes profits taking R, the rental rate of capital R, and $w_h(h)$, the wage of a worker with human capital h, as given. For expositional purposes, it will be easier to exploit our constant returns assumptions to aggregate over managers in (1) for all quantities.[47] As in the planner's problem, let $l_{iz}(s)$ denote the number of individuals with skill $s = (h, z)$ working as managers in sector i, then $Z_i \equiv \int z l_{iz}(s) ds$ is the total amount of manager skill in sector i. To this end, define

$$[l_{iz}(s), l_{ih}(s,j), K_{iz}, K_{ih}(j), Z_i, H_i(j), X_{iz}, X_{ih}, T_i(j)]$$

identically as in the main text, except now they are equilibrium allocations to solve for rather than the solution to the planner's problem. In what follows, we solve the problem of a representative sector i firm who uses technology (7).

Let c_z and $c_h(j)$ denote the unit cost of producing 1 unit of task z and task j output, respectively. Then the sector i firm's problem is

$$\max \left\{ p_i Y_i - c_z X_{iz} - \int_j^J c_h(j) T_i(j) dj \right\}, \quad \text{where}$$

$$c_z = \min \left\{ R K_{iz} + \int w_z Z_i \right\} \quad \text{s.t.} \quad X_{iz} = 1,$$

$$c_h(j) = \min \left\{ R K_{ih}(j) + \int w_h(h) l_{ih}(s,j) ds \right\} \quad \text{s.t.} \quad T_i(j) = 1.$$

In equilibrium, all firms maximize profits and capital and labor markets clear. Profit maximization implies

$$c_z = \left[\frac{\eta_i p_i Y_i}{X_{iz}} \right]^{\frac{1}{\omega}}, \quad c_h(j) = \left[\frac{(1-\eta_i) p_i Y_i}{X_{ih}} \right]^{\frac{1}{\omega}} \left[\frac{\nu_i(j) X_{ih}}{T_i(j)} \right]^{\frac{1}{\sigma}} \tag{34a}$$

and cost minimization implies that the amount of capital chosen for each task, and total units of managerial skill chosen for the managerial task, satisfy

$$\alpha c_z X_{iz} = R K_{iz}, \quad \alpha c_h(j) T_i(j) = R K_{ih}(j), \quad (1-\alpha) c_z X_{iz} = w_z Z_i. \tag{35a}$$

Similarly, the optimal choice of labor satisfies

$$(1-\alpha) c_h(j) T_i(j) \cdot b(h,j) \leq w_h(h) H_i(j) \quad \text{with equality if } l_{ih}(s,j) > 0. \tag{35b}$$

We already imposed market clearing of the sectoral goods in (7) and (24), that is, we used Y_i to denote both the supply and demand for sectoral goods. The equilibrium capital and labor market clearing conditions are identical to the resource constraints (5) and (6), respectively, where K is the exogenously given supply of capital and K_i are the sectoral supply and demand.

[47] While straightforward, solving for the aggregation yields several additional implicit prices at the indivdual manager level without adding any insights. Details are available upon request.

Definition 1 *A competitive equilibrium is a set of capital allocations* $\{K_{iz}, K_{ih}(j)\}$, *labor allocations* $\{l_{iz}(s), l_{ih}(s,j)\}$, *and prices* $\{p_i, w_z, w_h(h)\}$ *s.t.* (5)-(6), (24) *and* (33)-(35) *hold for* $i \in \{m, s\}$.

Most equations from (9) can be replicated from (35), with the additional step of plugging in the values for c_z and $c_h(j)$ at every step. Of course, the implicit cost of tasks can be of interest of themselves, but since we did not pursue this route in our main analysis, we focus on the planner's problem for ease of notation.

C Proofs

C.1 Proof of Lemma 1 and Corollary 1

The feasibility constraint (6) and existence of $\hat{h}(j)$ and $\hat{z}(j)$ imply that the number of people with skill s assigned to task j is

$$l_h(s,j)ds = \delta(j - \hat{j}(h)) \cdot \mathcal{I}\left[z \le \tilde{z}(h)\right] d\mu$$

where $\delta(\cdot)$ is the Dirac delta function and $\mathcal{I}$ the indicator function. Hence the allocation rule is completely determined by the assignment functions $\hat{h}(j)$ and $\hat{z}(j)$, and the productivity of all workers assigned to task $j = \hat{j}(h)$ is

$$H(j) = \int b(h, \hat{j}(h')) \cdot \delta(j - \hat{j}(h')) \cdot F(\tilde{z}(h')|h')dG(h').$$

With the change of variables $j' = \hat{j}(h')$, we can instead integrate over j':

$$\begin{aligned} H(j) &= \int b(\hat{h}(j'), j') \cdot \delta(j - j') \cdot F(\hat{z}(j')|\hat{h}(j'))g(\hat{h}(j')) \cdot \hat{h}'(j')dj' \\ &= b(\hat{h}(j), j) \cdot F(\hat{z}(j)|\hat{h}(j))g(\hat{h}(j)) \cdot \hat{h}'(j), \end{aligned}$$

which is (16).

For the optimal allocation, there can be no marginal gain from switching any worker's assignment. So for any $j' = j + dj$,

$$\begin{aligned} \frac{MPT_i(j) \cdot T_i(j)}{H_i(j)} \cdot b(\hat{h}(j), j) &\ge \frac{MPT_i(j') \cdot T_i(j')}{H_i(j')} \cdot b(\hat{h}(j), j'), \\ \frac{MPT_i(j') \cdot T_i(j')}{H_i(j')} \cdot b(\hat{h}(j'), j') &\ge \frac{MPT_i(j) \cdot T_i(j)}{H_i(j)} \cdot b(\hat{h}(j'), j), \end{aligned}$$

with equality if $|dj| = 0$. Substituting for $H_i(j) = H(j)/q_{ih}(j)$ using (16), we obtain

$$\frac{b(\hat{h}(j'), j')}{b(\hat{h}(j), j')} \ge \frac{\pi_{ih}(j')}{\pi_{ih}(j)} \cdot \frac{q_{ih}(j)F(\hat{z}(j)|\hat{h}(j))g(\hat{h}(j))\hat{h}'(j)}{q_{ih}(j')F(\hat{z}(j')|\hat{h}(j'))g(\hat{h}(j'))\hat{h}'(j')} \ge \frac{b(\hat{h}(j'), j)}{b(\hat{h}(j), j)},$$

and as $|dj| \to 0$,

$$\left[\partial \log b(\hat{h}(j), j)/\partial h\right] \cdot \hat{h}'(j) = d \log \left\{\pi_{ih}(j) \Big/ \left[q_{ih}(j)F(\hat{z}(j)|\hat{h}(j))g(\hat{h}(j))\hat{h}'(j)\right]\right\} \Big/ dj.$$

Now using the total derivative of $b(\hat{h}(j), j)$:

$$d \log b(\hat{h}(j), j)/dj = \left[\partial \log b(\hat{h}(j), j)/\partial h\right] \cdot \hat{h}'(j) + \partial \log b(\hat{h}(j), j)/\partial j, \tag{36}$$

and applying $\pi_{ih}(0) = 1$, we obtain (17):

$$H_i(j)/\pi_{ih}(j)H_i(0) = \exp\left[\int_0^j \frac{\partial \log b(\hat{h}(j'), j')}{\partial j'} dj'\right] \equiv B_j(j; \hat{h}). \tag{37}$$

Plugging (13) and (17) into (12) yields the first equality in (19) in the corollary, and note that (36) implies that $b(h, \hat{j}(h)) = B_h(h; \hat{j}) \cdot B_j(\hat{j}(h); \hat{h})$, which yields the second equality.

C.2 Proof of Proposition 1 and Corollary 2

First, we re-express all capital input ratios only in terms of the thresholds $[\hat{h}(j), \hat{z}]$. Plugging (17) into (9), and applying the task production function (8) we obtain

$$\pi_{ih}(j) = v_{ih}(j) \Big/ \left[\tilde{M}(j) B_j(j; \hat{h})^{1-\alpha}\right]^{1-\sigma}, \quad \text{where } v_{ih}(j) \equiv \frac{\nu_i(j)}{\nu_i(0)} \tag{38}$$

and $\tilde{M}(j) \equiv M(j)/M(0)$. Similarly, plugging (10) and (13) in (11) we obtain

$$\pi_{iz} = v_{iz} \cdot \left(\tilde{M}(z) \cdot \hat{z}^{1-\alpha}\right)^{\omega-1} \cdot \Pi_{ih}^{\frac{\sigma-\omega}{\sigma-1}}, \quad \text{where } v_{iz} \equiv \frac{\eta_i \nu_i(0)^{\frac{1-\omega}{\sigma-1}}}{1-\eta_i} \tag{39}$$

and $\tilde{M}(z) \equiv M(z)/M(0)$.

Now given a between-sector allocation rule $[q_{ih}(j), q_{iz}]$, the optimal within-sector allocation is described by $[\hat{h}(j)]_{j=0}^{J}$ that solves a fixed point defined by (16)-(17) in Lemma 1, and $\hat{z}$ that solves the fixed point defined by (13) and (39):

$$\hat{h}'(j) = \frac{H_i(0) \cdot v_{ih}(j)}{q_{ih}(j)} \Big/ \left\{[\tilde{M}(j) B_j(j)^{1-\alpha}]^{1-\sigma} B_h(\hat{h}(j)) F(\hat{z}(j)|\hat{h}(j)) g(\hat{h}(j))\right\} \tag{40a}$$

$$\hat{z}^{\alpha+\omega(1-\alpha)} = \frac{q_{iz}}{H_i(0) \cdot v_{iz}} \cdot \Pi_{ih}^{\frac{\sigma-\omega}{1-\sigma}} \cdot \tilde{M}(z)^{1-\omega} \cdot Z \tag{40b}$$

where the boundary conditions for the ODE in (40a) are $\hat{h}(0) = 0$ and $\hat{h}(J) = h_M$, which implies

$$H_i(0) \cdot \int v_{ih}(j) \left\{q_{ih}(j) \cdot [\tilde{M}(j) B_j(j)^{1-\alpha}]^{1-\sigma} B_h(\hat{h}(j)) F(\hat{z}(j)|\hat{h}(j)) g(\hat{h}(j))\right\}^{-1} dj$$
$$= h_M. \tag{40c}$$

The functions $[B_j(j), B_h(h), \hat{z}(j), \tilde{z}(h)]$, which represent relative wages in equilibrium, are defined in (15), (18) and (19); in particular, the first two are functions of $[\hat{h}(j), \hat{j}(h)]$ only. That is, system (40) is a fixed point only in terms of the thresholds, so their determination is independent of the total amount of physical capital and labor in either sector. All that matters is relative masses *across tasks*.

Existence of a fixed point is straightforward. For an arbitrary guess of $\hat{z}(j)$, Assumptions 1-2 imply existence of a solution to the differential equation (40a) by Picard-Lindelöf's existence theorem. Similarly, a solution to (40b) exists by Brouwer's fixed point theorem once we apply a minimum value for $\hat{z} \geq \underline{z} > 0$ such that the denominator does not converge to zero.

To show that the within-sector solution is unique, we need the following lemma:

Lemma 5 *Suppose $[q_h(j), q_z]$ are fixed and that $[\hat{h}(j), \hat{z}]$ and $[\hat{h}^1(j), \hat{z}^1]$ are both an equilibrium for one sector. For any connected subset $\mathcal{J}^1 \subseteq \mathcal{J}$, $\hat{h}$ and $\hat{h}^1$ can never coincide more than once on $\mathcal{J}^1$.*

Proof *We proceed by contradiction as in Lemmas 3-6 in Costinot and Vogel (2010). Suppose* **(i)** *$\hat{h}(j_a) = \hat{h}^1(j_a)$ and $\hat{h}(j_b) = \hat{h}^1(j_b)$ such that both $(j_a, j_b) \in \mathcal{J}^1$. Without loss of generality, we assume that $j_a < j_b$ are two adjacent crossing points. Then, since $[\hat{h}, \hat{h}^1]$ are Lipschitz continuous and strictly monotone in j, it must be the case that*

1. **(ii)** *$\hat{h}^{1\prime}(j_a) \geq \hat{h}'(j_a)$ and $\hat{h}^{1\prime}(j_b) \leq \hat{h}'(j_b)$; and* **(iii)** *$\hat{h}^1(j) > \hat{h}(j)$ for all $j \in (j_a, j_b)$; or*

2. **(ii)** *$\hat{h}^{1\prime}(j_a) \leq \hat{h}'(j_a)$ and $\hat{h}^{1\prime}(j_b) \geq \hat{h}'(j_b)$; and* **(iii)** *$\hat{h}^1(j) < \hat{h}(j)$ for all $j \in (j_a, j_b)$.*

Consider case **1**. *Condition* **(ii)** *implies*

$$\hat{h}^{1\prime}(j_b)/\hat{h}^{1\prime}(j_a) \leq \hat{h}'(j_b)/\hat{h}'(j_a)$$

so using (36)-(37) and (40a), and applying $\hat{h}^1(j) = \hat{h}(j)$ for $j \in \{j_a, j_b\}$ we obtain

$$\begin{aligned} 0 &< [\alpha + \sigma(1-\alpha)] \cdot \left[\int_{j_a}^{j_b} \frac{\partial \log b(\hat{h}^1(j'), j')}{\partial j'} dj' - \int_{j_a}^{j_b} \frac{\partial \log b(\hat{h}(j'), j')}{\partial j'} dj' \right] \\ &\leq \log \left[F(\hat{z}^1(j_b)|\hat{h}(j_b))/F(\hat{z}(j_b)|\hat{h}(j_b)) \right] - \log \left[F(\hat{z}^1(j_a)|\hat{h}(j_a))/F(\hat{z}(j_a)|\hat{h}(j_a)) \right] \end{aligned} \tag{41}$$

where the first inequality follows since (2), the log-supermodularity of b, implies

$$\partial \log b(h^1, j)/\partial j > \partial \log b(h, j)/\partial j \qquad \forall h^1 > h, \tag{42}$$

and applying **(iii)**. *Next, since (19) and Assumption 3.1 implies that $\hat{z}'(j) = \tilde{z}'(h)\hat{h}'(j) > 0$, Assumption 3.2 implies that the strict inequality in (41) holds only if*

$$\hat{z}^1(j_b)/\hat{z}(j_b) > \hat{z}^1(j_a)/\hat{z}(j_a) \quad \Leftrightarrow \quad \log \left[\tilde{z}^1(h_b)/\tilde{z}^1(h_a)\right] > \log \left[\tilde{z}(h_b)/\tilde{z}(h_a)\right]$$

where we have written $h_x \equiv \hat{h}(j_x)$ for $x \in \{a, b\}$. Plugging in for $\tilde{z}(\cdot)$ using (19) we obtain

$$\int_{h_a}^{h_b} \frac{\partial \log b(h', \hat{j}^1(h'))}{\partial h'} dh' > \int_{h_a}^{h_b} \frac{\partial \log b(h', \hat{j}(h'))}{\partial h'} dh'$$

and since $\hat{j}(h)$ is the inverse of $\hat{h}(j)$, **(iii)** *implies that $\hat{j}^1(h) < \hat{j}(h)$ for all $h \in (h_a, h_b)$. But (2), the log-supermodularity of b, implies*

$$\partial \log b(h, j^1)/\partial h < \partial \log b(h, j)/\partial h, \qquad \forall j^1 < j, \tag{43}$$

a contradiction. Case **2** *is symmetric.* ∎

Lemma 5 implies, in particular, that any within-sector equilibria must have identical $\hat{h}(j)$, since $\hat{h}(0) = 0$ and $\hat{h}(J) = h_M$ in all equilibria. Moreover, the lemma also implies that $\hat{h}(j)$ is determined independently of $\hat{z}$, which is uniquely determined by $\hat{h}(j)$ given (40). Hence, the within-sector equilibrium is unique.

Sectoral production function The corollary expresses sectoral output only in terms of sectoral capital and labor, and the optimal assignment rules. To derive this, first note that using (9)-(11), sectoral capital can be written as $K_i = K_i(0)\Pi_{K_i}$, which is the first equation in (21). Next, from (13), we know that Z_i is linear in $H_i(0)$:

$$Z_i = q_{iz} L_z \bar{z} = L_{iz}\bar{z} = H_i(0) \cdot \hat{z}\pi_{iz}, \quad \text{where} \quad L_z \equiv \int_{z > \tilde{z}(h)} d\mu, \quad \bar{z} = Z/L_z,$$

and using Lemma 1 and (38), so is total worker productivity:

$$\begin{aligned}\int \left[H_i(j) \middle/ b(\hat{h}(j), j)\right] dj &= \int q_{ih}(\hat{j}(h)) F(\tilde{z}(h)|h) g(h) dh \\ &= H_i(0) \cdot \int \left[\pi_{ih}(j)/B_h(\hat{h}(j))\right] dj \quad = L_i - L_{iz}.\end{aligned}$$

So rearranging, we can represent sectoral labor input as $L_i = H_i(0)\Pi_{L_i}$, which is the second equation in 21. Finally, use (8)-(11) to rewrite (7) as

$$Y_i = \psi_i \cdot \Pi_{K_i}^{\frac{\omega}{\omega-1}} \Pi_{ih}^{\frac{\sigma-\omega}{(\sigma-1)(1-\omega)}} M(0) K_i(0)^\alpha H_i(0)^{1-\alpha},$$

and replacing $[K_i(0), H_i(0)]$ with the expressions in (21) yields (20).

C.3 Proof of Theorem 1

Since Proposition 1 showed that the within-sector solution (and hence equilibrium) is unique, we only need to show that the sectoral allocation rules $\{[q_h(j)]_{j=0}^J, q_z\}$ are unique. In equilibrium, the allocation rules $[\hat{h}(j), \hat{z}]$ must be equal across sectors. Applying this to (40a) yields

$$q_h(j) = 1 \middle/ \left[1 + \frac{1 - q_h(0)}{q_h(0)} \cdot \frac{\nu_s(0)}{\nu_m(0)} \cdot \frac{\nu_m(j)}{\nu_s(j)}\right] \tag{44}$$

and $q_h(0)$ must solve (40c), so the dependence of the between-sector allocation rule on the within-sector rule comes only through $q_h(0)$. Likewise, the rule for splitting individuals between managers and workers, (40b), implies

$$q_z = 1 \middle/ \left[1 + \frac{1 - q_h(0)}{q_h(0)} \cdot \frac{\nu_s(0)}{\nu_m(0)} \cdot \frac{\eta_m(1-\eta_s)}{(1-\eta_m)\eta_s} \cdot \left(\frac{V_{sh}}{V_{mh}}\right)^{\frac{\sigma-\omega}{1-\sigma}}\right] \tag{45}$$

where V_{ih} is defined in (29) and depends on the within-sector allocation rule through B_j. But note that given $q_h(0)$, the other $q_h(j)$ only depend on the task intensity parameters $\nu_i(j)$ and are uniquely fixed by (44). Then we know from Proposition 1 that all $\hat{h}(j)$ are uniquely determined, as well as $\hat{z}$. Hence, q_z also only depends on the manager intensity parameters η_i, and are uniquely determined by (45) given $q_h(0)$.

So in equilibrium, $q_h(0)$ alone must solve the implied sectoral shares in (22) given (20):

$$\begin{aligned}&\frac{q_h(0)}{1 - q_h(0)} \equiv Q(q_h(0)) \\ =&\frac{\gamma_s}{\gamma_m} \cdot \left(\frac{\psi_s}{\psi_m}\right)^{\epsilon-1} \cdot \left(\frac{\Pi_{sh}}{\Pi_{mh}}\right)^{\frac{(\sigma-\omega)(1-\epsilon)}{(1-\sigma)(1-\omega)}} \cdot \left(\frac{\Pi_{K_s}}{\Pi_{K_m}}\right)^{\left(\alpha+\frac{\omega}{1-\omega}\right)(1-\epsilon)} \cdot \left(\frac{\Pi_{L_s}}{\Pi_{L_m}}\right)^{-[\alpha+\epsilon(1-\alpha)]}.\end{aligned} \tag{46}$$

Existence of a solution is straightforward, since the LHS of (46) increases smoothly from 0 to ∞ as $q_h(0)$ varies from 0 to 1, while the RHS is always positive and strictly bounded regardless of the value of $q_h(0)$. To show uniqueness then, it suffices to show that the RHS cannot cross LHS more than once. We will consider the log derivatives of the RHS of (46) term by term.

Let Δ_x denote the log-derivative of x w.r.t. $q_h(0)$. Since Assumption 5 implies that

$$\Delta_{B_j(j)} = \int_0^j \frac{\partial^2 \log b(\hat{h}(j'), j')}{\partial h \partial j'} \cdot \frac{d\hat{h}(j')}{dj'} \cdot dj' < \varepsilon \tag{47}$$

for all $\varepsilon > 0$, we obtain from (38) that

$$\Delta_{\pi_{ih}} = (1-\alpha)(\sigma-1) \cdot \Delta_{B_j(j)} \approx 0$$

so $\Delta_{\Pi_{ih}} \approx 0$. Likewise, Assumption 5 also implies that

$$\Delta_{B_h(h)} = \int_0^h \frac{\partial^2 \log b(h', \hat{j}(h'))}{\partial h' \partial j} \cdot \frac{d\hat{j}(h')}{dh'} \cdot dh' < \varepsilon \tag{48}$$

for all $\varepsilon > 0$. This implies that $\hat{h}(j)$ is not affected by the choice of $q_h(0)$, and it is independent of the determination of $\hat{z}$ by Lemma 5. Intuitively, Assumption 5 makes the model behave as if there were no log-supermodularity. Then since we assume a constant returns technology, all worker allocations approach constant multiples of H_0 and does not depend on its particular value. So $\Delta_{\Pi_{ih}} \approx 0$, and $\Delta_{\Pi_{K_i}}$ only depends on $\Delta_{\hat{z}}$ since from the definition of Π_{K_i} in (21) and (39),

$$\Delta_{\pi_{iz}} = (1-\alpha)(\omega-1)\Delta_{\hat{z}} \quad \Rightarrow \quad \Delta_{\Pi_{K_i}} \Pi_{K_i} = \pi_{iz} \cdot (1-\alpha)(\omega-1)\Delta_{\hat{z}},$$

Similarly, $\Delta_{\Pi_{L_i}}$ only depends on $\Delta_{\hat{z}}$ as well, since from (19) and (48) we obtain

$$\Delta_{\tilde{z}(h)} = \Delta_{\hat{z}} + \Delta_{B_h(h)} \approx \Delta_{\hat{z}}. \tag{49}$$

so using Leibniz' rule,

$$\begin{aligned} \Delta_Z \cdot Z &= -\Delta_{\hat{z}} \cdot \int \left[\tilde{z}(h)^2 \cdot f(\tilde{z}(h)|h)\right] g(h)dh, \\ \Delta_{L_z} \cdot L_z &= -\Delta_{\hat{z}} \cdot \int \left[\tilde{z}(h) \cdot f(\tilde{z}(h)|h)\right] g(h)dh, \\ \Rightarrow \quad \Delta_{\bar{z}} &= \Delta_Z - \Delta_{L_z} = \Delta_{\hat{z}} \cdot \underbrace{\int \left\{\tilde{z}(h)\left[1/L_z - \tilde{z}(h)/Z\right] \cdot f(\tilde{z}(h)|h)\right\} g(h)dh}_{\equiv \Lambda \in (0,1)} \end{aligned} \tag{50}$$

where the inequality follows from selection and Assumption 4.1, so using this and (48), from the definition of Π_{L_i} in (21) we obtain

$$\Delta_{\Pi_{L_i}} \Pi_{L_i} = (\hat{z}/\bar{z})\pi_{iz} \cdot \left[\alpha + \omega(1-\alpha) - \Lambda\right] \Delta_{\hat{z}}.$$

Now rearranging (40b), plugging in (50), and using (40a) at $j = 0$ we obtain

$$\begin{aligned} &\left\{\alpha + \omega(1-\alpha) + \hat{z} f(\hat{z}|0)/F(\hat{z}|0) + \int \left[\tilde{z}(h)^2 \cdot f(\tilde{z}(h)|h)\right] g(h)dh\right\} \Delta_{\hat{z}} \\ &= \Delta_{q_z} - 1 \equiv \Gamma(X), \end{aligned}$$

since $H_s(0) = q_h(0)H(0)$, $\Delta_{\hat{h}'(0)} = 0$ as it does not vary with $q_h(0)$, and $\Gamma(X)$ is defined from (45):

$$\Gamma(X) = q_h(0)(X-1)/\left[q_h(0) + (1-q_h(0))X\right],$$
$$\text{where} \quad X \equiv \frac{\nu_s(0)}{\nu_m(0)} \cdot \frac{\eta_m(1-\eta_s)}{(1-\eta_m)\eta_s} \cdot \left(\frac{V_{sh}}{V_{mh}}\right)^{\frac{\sigma-\omega}{1-\sigma}}.$$

So it follows that the log-slope of the RHS in (46) is

$$-\Bigg\{(1-\epsilon)(1-\alpha)[\alpha+\omega(1-\alpha)] \cdot \left[\frac{\pi_{sz}}{\Pi_{K_s}} - \frac{\pi_{mz}}{\Pi_{K_m}}\right]$$
$$+ (\hat{z}/\bar{z})[\alpha+\epsilon(1-\alpha)][\alpha+\omega(1-\alpha)-\Lambda] \cdot \left[\frac{\pi_{sz}}{\Pi_{L_s}} - \frac{\pi_{mz}}{\Pi_{L_m}}\right]\Bigg\}$$
$$\times \frac{\Gamma(X)}{\alpha+\omega(1-\alpha)+\hat{z}f(\hat{z}|0)/F(\hat{z}|0) + \int\left[\tilde{z}(h)^2 \cdot f(\tilde{z}(h)|h)\right]g(h)dh}.$$

The log-slope of the LHS in (46) is $1/\left[1-q_h(0)\right]$, which increases from 1 to ∞ as $q_h(0)$ increases from 0 to 1, and is larger than $\Gamma(X)$ for all $X > 0$. Hence it suffices to show that the absolute value of all terms multiplying $\Gamma(X)$ are less than 1, which is true in particular due to Assumption 4.2.

Intuitively, what the planner cares about is the marginal products of Z and H in total. So when the distribution of z has a fat tail, the response of $\hat{z}$ to the choice of $q_h(0)$ is minimal as it changes Z smoothly along its entire support.

C.4 Proof of Proposition 2

Part 1. By Lemma 5, we know that no crossing can occur on $(0, \underline{j})$ or $(\bar{j}, J)$, since $\hat{h}$ and $\hat{h}^1$ already coincide at the boundaries 0 and J. Similarly, we also know from Theorem 1 that it can never be the case that there is no crossing ($\hat{h}^1(j) > \hat{h}(j)$ or $\hat{h}^1(j) < \hat{h}(j)$ for all $j \in \mathcal{J} \setminus \{0, J\}$). Hence, there must be a single crossing in $\mathcal{J}^1$ since Lemma 5 also rules out multiple crossings.

At this point, the only possibility for j^* not to exist is if instead, there exists a single crossing j^{**} such that **(i)** $\hat{h}^1(j) < \hat{h}(j)$ for all $j \in (0, j^{**})$ and **(ii)** $\hat{h}^1(j) > \hat{h}(j)$ for all $j \in (j^{**}, J)$. If so, since $[\hat{h}, \hat{h}^1]$ are Lipschitz continuous and strictly monotone in j, it must be the case that $\hat{h}^{1\prime}(0) < \hat{h}'(0)$, $\hat{h}^{1\prime}(j^{**}) > \hat{h}'(j^{**})$ and $\hat{h}^{1\prime}(J) < \hat{h}'(J)$. This implies

$$\hat{h}^{1\prime}(j^{**})/\hat{h}^{1\prime}(0) \geq \hat{h}'(j^{**})/\hat{h}'(0), \qquad \hat{h}^{1\prime}(J)/\hat{h}^{1\prime}(j^{**}) \leq \hat{h}'(J)/\hat{h}'(j^{**}). \tag{51}$$

Let us focus on the first inequality. Using (37) and (40a) we obtain

$$\begin{aligned} 0 > &[\alpha+\sigma(1-\alpha)] \cdot \left[\int_0^{j^{**}} \frac{\partial \log b(\hat{h}^1(j), j)}{\partial j} dj - \int_0^{j^{**}} \frac{\partial \log b(\hat{h}(j), j)}{\partial j} dj\right] \\ \geq &(1-\sigma)m + \log\left[F(\hat{z}^1(j^{**})|\hat{h}(j^{**}))/F(\hat{z}(j^{**})|\hat{h}(j^{**}))\right] - \log\left[F(\hat{z}^1(0)|\hat{h}(0))/F(\hat{z}(0)|\hat{h}(0))\right]. \end{aligned} \tag{52}$$

where the first inequality follows from (42), and applying **(i)**. Since $m > 0$, if $\sigma \in (0, 1)$, Assumption 3 implies that the strict inequality in (52) holds only if

$$\int_0^{h^{**}} \frac{\partial \log b(h', \hat{j}^1(h'))}{\partial h'} dh' < \int_0^{h^{**}} \frac{\partial \log b(h', \hat{j}(h'))}{\partial h'} dh'$$

where we have written $h^{**} \equiv \hat{h}(j^{**})$. And since $\hat{j}(h)$ is the inverse of $\hat{h}(j)$, **(i)** implies that $\hat{j}^1(h) > \hat{j}(h)$ for all $h \in (0, h^{**})$. But this violates (43), the log-supermodularity of b. The case for the second inequality in (51) is symmetric.

Part 2. Let Δ_x denote the log-derivative of x w.r.t. $\tilde{m}$. Applying (38) into the definition of Π_{ih} in (10), we obtain

$$\begin{aligned} \Delta_{\Pi_{ih}} \cdot \Pi_{ih} &= (\sigma - 1) \int_{\underline{j}}^{\bar{j}} \pi_{ih}(j) dj + \int \left\{ \pi_{ih}(j) \cdot (1-\alpha)(\sigma - 1) \cdot \Delta_{B_j(j)} \right\} dj \\ &\approx (\sigma - 1) \int_{\underline{j}}^{\bar{j}} \pi_{ih}(j) dj \end{aligned} \tag{53}$$

where the approximation follows from Assumption 5 and (47). Hence $\Delta_{\Pi_{ih}} < 0$ if $\sigma < 1$. Rearranging (40b) and using (40a) at $j = 0$ we obtain

$$0 > \frac{\sigma - \omega}{1 - \sigma} \cdot \Delta_{\Pi_{ih}} - \Delta_{\hat{h}'(0)} = \left[\alpha + \omega(1-\alpha) + \hat{z} f(\hat{z}|0)/F(\hat{z}|0) \right] \Delta_{\hat{z}} - \Delta_Z \tag{54}$$

where the inequality holds if $\omega < \sigma < 1$, and since we know from part 1 that $\Delta_{\hat{h}'(0)} \geq 0$. Now suppose $\Delta_{\hat{z}} \geq 0$. Then for (54) to hold it must be the case that $\Delta_Z > 0$, but from (50), $\Delta_Z \leq 0$ if $\Delta_{\hat{z}} \geq 0$, a contradiction. Hence, $\hat{z}^1 < \hat{z}$, and $\tilde{z}^1(h) < \tilde{z}(h)$ for all h by (49).

C.5 Proof of Lemmas 2 and 3

From (28), the $\Delta_{V_i(j)}$'s are sector-neutral and common across sectors, except for $\Delta_{V_i(z)}$. Under Assumption 5, (47)-(48) imply

$$\Delta_{V_i(j)} = \sigma - 1 < 0 \quad \forall j \in \mathcal{J}^1 \quad \text{and 0 otherwise.} \tag{55a}$$

So for workers, any difference in how the share of task j employment evolves differentially across sectors depends only on $\Delta_{V_{L_i}}$, the sum of within-sector employment shifts, weighted by the employment shares of all tasks within a sector $V_i(j)/V_{L_i} = L_i(j)/L_i$. Since we know that intermediate jobs are the ones that are declining, from the definition of Π_{L_i} in (21) a measure of the speed of polarization among workers is the total change in their employment:

$$\Delta_{V_{il}} V_{il} = \int_{\mathcal{J}} V_i(j) \cdot \Delta_{V_i(j)} dj = (\sigma - 1) \cdot \int_{\underline{j}}^{\bar{j}} V_i(j) dj$$

and we have used (55a). So we can compare the speeds of polarization across the two sectors from

$$\Delta_{V_{ml}} - \Delta_{V_{sl}} = (\sigma - 1) \cdot \int_{\underline{j}}^{\bar{j}} \left\{ \left[\frac{\nu_m(j)}{V_{ml}} - \frac{\nu_s(j)}{V_{sl}} \right] \cdot \left[M(j) B_j(j)^{1-\alpha} \right]^{\sigma-1} \Big/ B_h(\hat{h}(j)) \right\} dj. \tag{55b}$$

Manager employment has sector-differential effects through V_{ih}: Under Assumption 5 and using (53), we obtain

$$\Delta_{V_m(z)} - \Delta_{V_s(z)} = (\sigma - \omega) \cdot \int_{\underline{j}}^{\bar{j}} \left\{ \left[\frac{\nu_m(j)}{V_{mh}} - \frac{\nu_s(j)}{V_{sh}} \right] \cdot \left[M(j) B_j(j)^{1-\alpha} \right]^{\sigma - 1} \right\} dj. \tag{55c}$$

Equations (55b) and (55c) imply that a sufficient condition for both horizontal and vertical polarization to be faster in manufacturing, as in the data, is $\omega < \sigma < 1$ and $\nu_m(j) \gg \nu_s(j)$ for all $j \in \mathcal{J}^1$, which is Lemma 2.

Structural change From (23) and (46) we obtain

$$\Delta_{L_s} = L_m \cdot \left\{ \Delta_{V_{L_s}} - \Delta_{V_{L_m}} + \Delta_Q \right\}. \tag{56a}$$

The term $\Delta_{L_s} - \Delta_{L_m}$ is the first-order force of structural change that comes only from the change in selection rules. However, since this takes us off the between-sector equilibrium, $q_h(0)$ must shift to satisfy the equilibrium condition (46). The net amount of structural change will depend on whether the selection effect is overturned or reinforced by the change in $q_h(0)$.

Since $Q(q_h(0))$ in (46) changes monotonically from 0 to ∞ in $q_h(0)$, we only need to consider the direction of the change of the RHS off equilibrium. Using (55), the log-derivative of the RHS of (46) can be written as

$$\begin{aligned} &(1-\epsilon) \left\{ \frac{\sigma - \omega}{(1-\sigma)(1-\omega)} \cdot \left(\Delta_{V_{sh}} - \Delta_{V_{mh}} \right) + \left(\alpha + \frac{\omega}{1-\omega} \right) \left(\Delta_{\Pi_{K_s}} - \Delta_{\Pi_{K_m}} \right) \right\} \\ &- \left[\alpha + \epsilon (1-\alpha) \right] \left(\Delta_{V_{L_s}} - \Delta_{V_{L_m}} \right). \end{aligned} \tag{56b}$$

Under Lemma 2, the part with $\Delta_{V_{ih}}$'s is positive from (55b). The part with $\Delta_{\Pi_{K_i}}$ is determined by

$$\Delta_{\Pi_{K_i}} \Pi_{K_i} = \Pi_{ih} \Delta_{V_{ih}} + \pi_{iz} \Delta_{\pi_{iz}}, \tag{56c}$$

$$\Delta_{\pi_{sz}} - \Delta_{\pi_{mz}} = \frac{\sigma - \omega}{1 - \sigma} \left(\Delta_{V_{sh}} - \Delta_{V_{mh}} \right). \tag{56d}$$

Clearly, capital polarizes along with labor, both horizontally and vertically; and the speed is faster in manufacturing if the assumptions in Lemma 3 holds.

Why structural change cannot be overturned, as explained in the text, is also formalized here: Even if there is a decline in $q_h(0)$ due to the negative effect coming from last term in (56b) dominating the positive effect from the first two terms, it can never overturn the direction of structural change in (56a) as long as $\epsilon < 1$. Equations (55)-(56) also make it clear that structural change depends differently on the productivities of capital and labor.

D Quantitative Model and Numerical Details

With discrete tasks, it must be that the marginal product of the threshold worker is equalized between tasks:

$$MPT_{i0} \cdot \frac{(1-\alpha) T_{i0}}{L_{i0}} = MPT_{i1} \cdot \frac{(1-\alpha) T_{i1}}{\bar{h}_1 L_{i1}} \cdot \hat{h}_1,$$

$$MPT_{ij} \cdot \frac{(1-\alpha)T_{ij}}{(\bar{h}_j - \chi_j)L_{ij}} \cdot (\hat{h}_{j+1} - \chi_j) = MPT_{ij+1} \cdot \frac{(1-\alpha)T_{i,j+1}}{(\bar{h}_{j+1} - \chi_{j+1})L_{i,j+1}} \cdot (\hat{h}_{j+1} - \chi_{j+1})$$

using Assumption 2, and L_{ij} is the measure of workers in sector i, task j and $\bar{h}_j \equiv H_{ij}/L_{ij}$. Thus, we are assuming that the means of skills in task j are equal across sectors $i \in \{m, s\}$, which is true when tasks are a continuum. Then

$$\hat{h}_1 = \frac{\bar{h}_1 L_{i1}}{\pi_{i1} L_{i0}}, \qquad \frac{\hat{h}_{j+1} - \chi_{j+1}}{\hat{h}_{j+1} - \chi_j} = \frac{\pi_{ij}(\bar{h}_{j+1} - \chi_{j+1})L_{i2}}{\pi_{i,j+1}(\bar{h}_j - \chi_j)L_{i1}}, \tag{57}$$

where π_{ij} is the discrete version of (9), and can be expressed using (57) as

$$\pi_{i1} = \frac{\nu_{i1}}{\nu_{i0}} \cdot \left(\frac{M_1}{M_0} \cdot \hat{h}_1^{1-\alpha}\right)^{\sigma-1}, \qquad \frac{\pi_{i,j+1}}{\pi_{ij}} = \frac{\nu_{i2}}{\nu_{i1}} \cdot \left[\frac{M_{j+1}}{M_j}\left(\frac{\hat{h}_{j+1} - \chi_{j+1}}{\hat{h}_{j+1} - \chi_j}\right)^{1-\alpha}\right]^{\sigma-1}. \tag{58}$$

In equilibrium, indifference across tasks for threshold workers imply

$$\begin{aligned} & w_0 = w_z \hat{z} = w_1 \hat{h}_1, \quad w_j(\hat{h}_{j+1} - \chi_j) = w_{j+1}(\hat{h}_{j+1} - \chi_{j+1}) \\ \Rightarrow \quad & w_z/w_0 = 1/\hat{z}, \quad w_1/w_0 = 1/\hat{h}_1, \quad w_{j+1}/w_j = \frac{\hat{h}_{j+1} - \chi_{j+1}}{\hat{h}_{j+1} - \chi_j}. \end{aligned} \tag{59}$$

which is used to calibrate the distribution of skills in Section 4.3. The rest of the parameters are calibrated as follows:

1. Guess (σ, ω).
2. Given elasticities, first fit 1980 moments:
   (a) Guess (M, A_m).
   (b) Plug in the threshold values $\mathbf{x}_{1980}$ implied by the skill distribution, along with the empirical values of $(L_{iz}, L_{i0}, \ldots, L_{i9})$, the employment shares of each occupation in sector $i \in \{m, s\}$ from Table 5, into (13) and (57). Then we recover all the ν_{ij}'s from (57)-(58), and the η_i's from (13) and (39) in closed form (since $M_j = M$ are assumed to be equal for all j). This ensures that the 1980 equilibrium exactly fits within-sector employment shares by occupation (20 parameters, 20 moments).
   (c) Repeat from (a) until we exactly fit the manufacturing employment share in 1980, and output per worker of 1.[48] Since (20) and (22) are monotone in (M, A_m), the solution is unique (2 parameters, 2 moments).
3. Given elasticities and all parameters, calibrate growth rates to 2010 moments:
   (a) Guess m_0.
   (b) Guess $\{m_j\}_{j=z,1}^{9}$. Plug in threshold values $\mathbf{x}_{2010}$ and new TFP's into (13) and (57), which yields equilibrium employment shares by occupation, within each sector. Then use (20)-(22) to solve for the 2010 equilibrium, which yields equilibrium employment shares between sectors.

[48]The latter must be matched since the value of K_{1980} we plug in from the data was normalized by 1980's output.

(c) Repeat from (a) until we exactly exactly fit aggregate GDP (or equivalently TFP) in 2010. (1 parameter, 1 moment).

4. Repeat from 1. to minimize the distance between the within-sector employment shares by occupation (but not necessarily by sector) implied by the 2010 model equilibrium and the data (13 parameters, 21 moments).

E Tables and Figures Not in Text

Ranked by mean wage	COC	Employment Shares (%)				Rel. Wages	
(except management)	Group	1980	2010	Manufacturing		1980	2010
Low Skill Services	400	**10.44**	**13.92**	**0.59**	**0.23**	**0.65**	**0.55**
Middle Skill		**59.09**	**46.48**	**25.86**	**12.93**	**0.90**	**0.77**
Administrative Support	300	16.57	14.13	3.47	1.53	0.78	0.68
Machine Operators	700	9.81	3.75	8.79	3.02	0.84	0.64
Transportation	800	8.73	6.64	3.80	2.28	0.89	0.63
Sales	240	7.87	9.37	0.79	0.62	0.94	0.90
Technicians	200	3.23	3.86	1.00	0.57	1.04	1.12
Mechanics & Construction	500	7.91	6.02	4.44	3.19	1.06	0.81
Miners & Precision Workers	600	4.97	2.71	3.58	1.73	1.09	0.77
High Skill		**19.22**	**26.16**	**3.87**	**3.64**	**1.26**	**1.30**
Professionals	40	11.02	16.51	1.73	1.45	1.21	1.26
Management Support	20	8.20	9.65	2.14	2.20	1.32	1.37
Management	1	**11.26**	**13.44**	**2.47**	**2.59**	**1.39**	**1.68**

Table 5: Occupation×Sector Employment and Relative Wages

Source: US Census (5%), 1980 and 2010. All employment shares are in percent of aggregate employment. The first two columns show the employment share of each occupation for each year. The "Manufacturing" columns show manufacturing employment of each occupation for each year (so the sum across all occupations is the manufacturing employment share). Relative wages are normalized so that the mean wage across all occupations is 1.

Ranked by mean wage (except management)	(1) m (%)	(2) m_j (%)	BM m_j (%)	Data
Low Skill Services	**1.973**	**-2.726**	**-0.731**	
Middle Skill				
Administrative Support	1.973	1.252	2.930	
Machine Operators	1.973	10.018	9.122	
Transportation	1.973	3.326	4.348	
Sales	1.973	-1.895	0.012	
Technicians	1.973	-2.484	-1.144	
Mechanics & Construction	1.973	1.742	2.315	
Miners & Precision Workers	1.973	6.367	6.328	
High Skill				
Professionals	1.973	-3.973	-2.248	
Management Support	1.973	-1.973	-0.489	
Management	**1.973**	**-1.438**	**-0.017**	
Aggregate TFP growth (%)	**1.030**	**1.030**	**1.030**	**1.030**
a_m (Manu TFP growth, %)	0.252	0.252	2.943	2.229
a_s (Serv TFP growth, %)	-1.205	2.021	0.308	0.743

Table 6: Recalibrated TFP Growth Rates for Counterfactuals

Column (1) stands for the counterfactual in which we set $m_j = m$ and calibrate (a_m, a_s) to match sectoral TFP's, and (2) for when we let $(\{m_j\}_{j=z,0}^{9}, a_m, a_s)$ all vary simultaneously. "BM" stands for the benchmark calibration. For all scenarios, aggregate GDP growth (and consequently TFP growth) is matched exactly, shown in the first row of the bottom panel. For the "BM" and "Data" columns, the a_m and a_s rows show the empirical growth rates of the manufacturing and services sectors' TFP's, respectively.

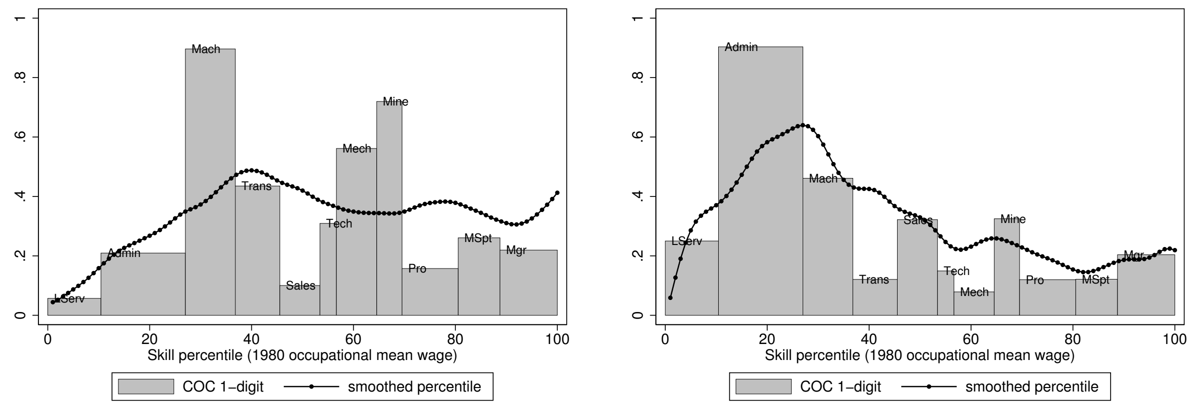
	ΔL_j
TFP	- 9.584 ***
R^2	0.939

RTI (DOT)	0.429						
	(0.268)						
Routine manual		0.797**		0.618			
		(0.206)		(0.527)			
Manual interpersonal			-0.767**	-0.192			
			(0.192)	(0.549)			
College share 1980					-11.142*		-7.994**
					(3.599)		(2.269)
ΔCollege share 1980-2010						-33.673*	-20.295*
						(17.410)	(13.547)
Constant	1.061	0.377	3.281**	1.065	4.035*	4.818*	5.204*
	(0.941)	(0.738)	(0.970)	(2.339)	(1.401)	(1.674)	(1.759)
R^2	0.184	0.635	0.588	0.640	0.439	0.372	0.539

Standard errors in parentheses, $^{\dagger}p < 0.10$, $^{*}p < 0.05$, $^{**}p < 0.01$

Table 7: Task-Specific TFP Growth, Employment, and Empirical Measures

The first panel shows the results from regressing employment share changes on the calibrated task-specific TFP growth rates, m_j. The second panel shows the results from regressing the TFP growth rates on various occupation-level empirical measures.

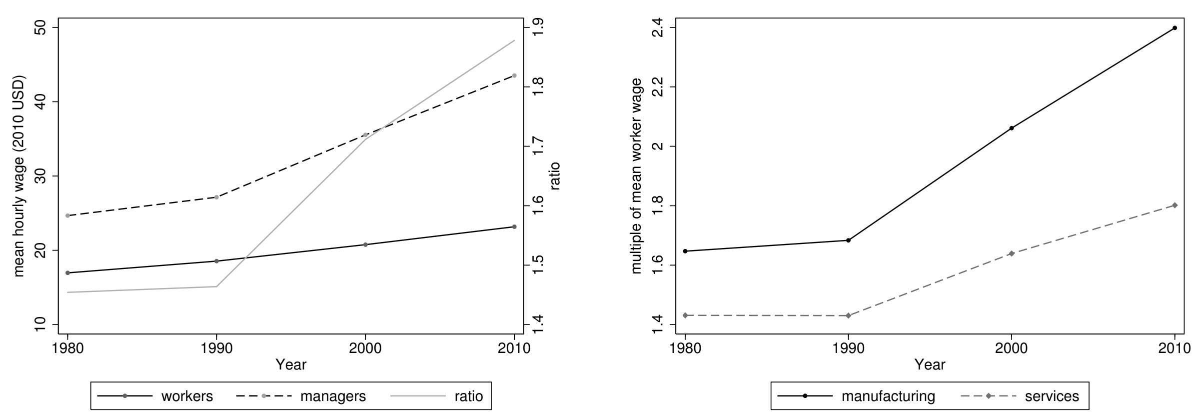


(a) Manufacturing

(b) Routinization

Fig. 20: Manufacturing Employment Shares and Routine Job Shares

Source: U.S. Census (5%). Left: Manufacturing employment share by occupation-skill percentile in 1980. Right: Share of top employment-weighted third of occupations in terms of RTI by skill percentile, replicates Autor and Dorn (2013) who construct RTI from detailed task requirements by occupation in DOT. Occupations are ranked by their 1980 mean wage for 11 one-digit groups and smoothed across 322 three-digit groups, separately. The x-axis units are in percent share of employment. Further details in text and Appendix A.

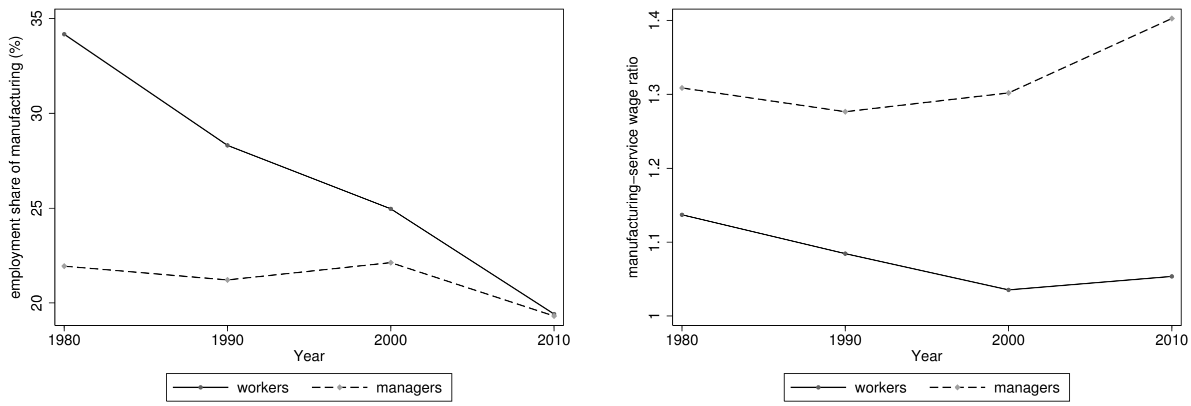


(a) Aggregate Wage Levels and Ratio

(b) Manager Wage by Sector

Fig. 21: Relative Manager Wages

Source: U.S. Census (5%). Left: levels and ratio of mean wages or managers and all other workers in aggregate. Right: relative mean wage of managers over all other workers within manufacturing and services. "Manufacturing" combines manufacturing, mining and construction, and services subsumes service and government. See Appendix A for how we define management in the census and Figure 19 for a detailed breakdown of the manager group.



(a) Manufacturing Employment Share (b) Manufacturing-Services Average Wage Ratio

Fig. 22: Manufacturing vs. Services by Occuaption

Source: U.S. Census (5%). Left: manufacturing employment share within the manager occupation group and all other workers. Right: mean wage of manufacturing employment relative to services employment within the manager occupation group and all other workers. "Manufacturing" combines manufacturing, mining and construction, and services subsumes service and government. See Appendix A for how we define management in the census and Figure 19 for a detailed breakdown of the manager group.

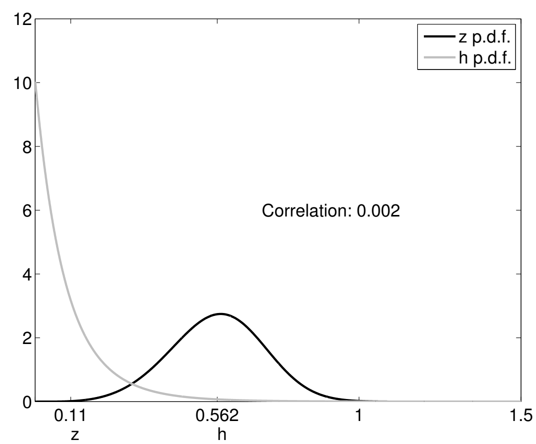
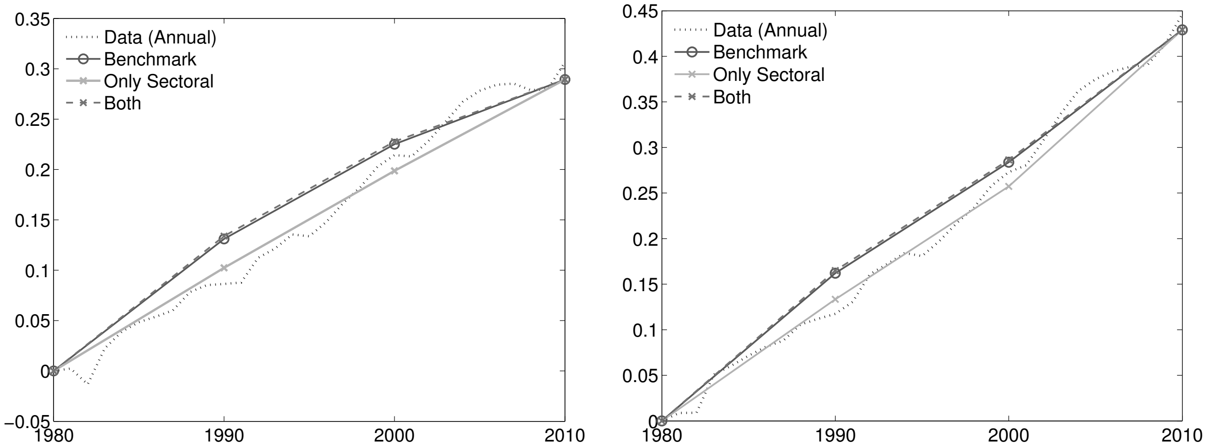


Fig. 23: Calibrated Skill Distribution

We use a type IV bivariate Pareto distribution to model the distribution over worker and manager skills (h, z). The figure depicts the marginal distributions of each skill, and also their mean below the x-axis.

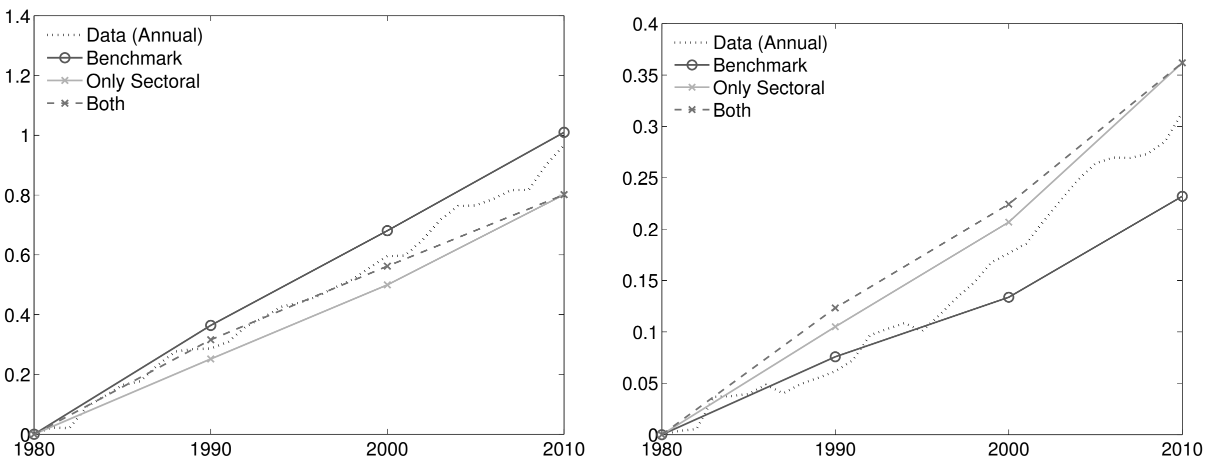


(a) Log TFP Growth

(b) Log GDP per Worker Growth

Fig. 24: Aggregate Output and TFP Growth

Data: NIPA. Log 1980 levels are normalized to 0, so the slopes of the lines are the growth rates.

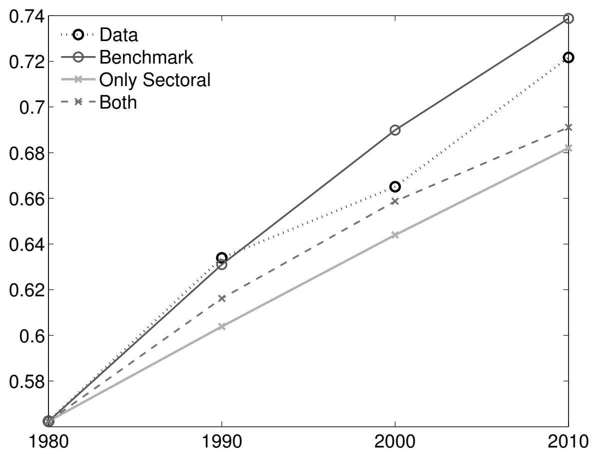


(a) Log GDP per Worker, Manufacturing

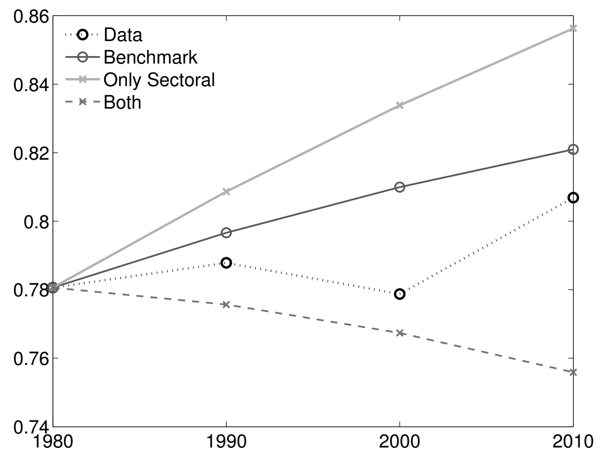
(b) Log GDP per Worker, Services

Fig. 25: Benchmark vs. Counterfactuals, GDP per Worker

Data: NIPA. "Manufacturing" combines manufacturing, mining and construction, and services subsumes service and government. Log 1980 levels are normalized to 0, so the slopes of the lines are the growth rates.



(a) Services Employment Share, Routine Jobs



(b) Services Employment Share, Managers

Fig. 26: Benchmark vs. Counterfactuals, Structural Change

Vertical axes are the fractions of routine jobs (left) and mangers (right) in services.

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