The macroeconomic cost of climate volatility*

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Abstract

We study the impact of climate volatility on economic growth exploiting data on 133 countries between 1960 and 2005. We show that the conditional (*ex ante*) volatility of annual temperatures increased steadily over time, rendering climate conditions less predictable across countries, with important implications for growth. Controlling for concomitant changes in temperatures, a +1°C increase in temperature volatility causes on average a 0.9 per cent decline in GDP growth and a 1.3 per cent increase in the volatility of GDP. Unlike changes in average temperatures, changes in temperature volatility affect both rich and poor countries.

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1 Introduction

Rising temperatures are known to have a negative impact on economic growth, particularly in poor countries. This paper shows that climate change also affects economic outcomes through a volatility channel. Using the dataset of Dell et al. (2012) and a panel VAR with stochastic volatility, we estimate the conditional volatility of average annual temperatures for 133 countries between 1961 and 2005. These estimates measure the volatility of the residual component of annual temperatures that cannot be forecasted using past data, quantifying the *ex-ante* 'temperature risk' faced by households and firms in a given country at a given point in time. The model captures the interaction between levels and variances of annual temperatures and GDP growth rates, allowing the identification of exogenous temperature volatility shocks. Our analysis yields two main results. The first one is that temperature volatility increased steadily over time, even in regions that were only marginally affected by global warming. The second one is that temperature volatility matters for economic activity. Controlling for temperature levels, a 1°C increase in volatility causes on average a 0.9 per cent decline in GDP growth and a 1.3 per cent increase in the volatility of the GDP growth rate. In other words, volatile temperatures lead at once to lower and more variable income growth. These dynamics affect rich, non-agricultural countries too, and they are not driven by the occurrence of large fluctuations in GDP, temperatures or precipitations. These findings demonstrate that risk plays an important role in the nexus between climate and the economy. Economic agents respond to changes in the expected variability of the environment, and, as in other macro-financial contexts, lower predictability is by itself detrimental for growth. This suggests that climate risk may have important *ex-ante* implications for welfare, as uncertainty affects the economy before and independently of any actual change in temperatures.

Related literature. Our work lies at the intersection of two strands of research. The first one studies the economic implications of climate change. The negative influence of global warming on income and welfare was originally highlighted using reduced-form Integrated Assessment Models (IAMs), and more recently confirmed by general equilibrium models of the interaction between climate and the economy. [1] A large body of empirical evidence documents the relation between weather outcomes and productivity, output and economic growth, as well as political stability, migration or mortality (Dell et al., 2009; Dell et al., 2014). Although researchers broadly agree that rising temperatures reduce growth in poor countries, the evidence on developed economies is more mixed (Burke and Leigh, 2010, Dell et al., 2012). The ambiguity also arises in studies that focus on

[1] See respectively Tol (2009), Nordhaus and Moffat (2017), Stern (2016) and Golosov et al. (2014), Hassler et al. (2016), Acemoglu et al. (2012).

agriculture: the negative influence of higher temperatures in EMEs is uncontroversial (Dell et al., 2014), while studies based on within-country variability in the USA reach conflicting conclusions (Deschênes and Greenstone, 2007; Fisher et al., 2012). We exploit the dataset of Dell et al. (2012) (henceforth DJO) to document a new "volatility" effect of climate change that operates over and above the "level" effect studied in previous contributions. Our results suggest that temperature volatility affects growth in developed economies too. The second strand of research examines the macroeconomic implications of changes in risk and uncertainty. The relevance of macro-financial volatility for consumption, investment and production is well documented in the literature.[2] By studying the impact of temperature volatility on growth we document a new, thus-far ignored source of aggregate risk for the business cycle.

We are aware of two existing studies of the linkages between climate volatility and growth. Donadelli et al. (2020) examine the relation between annual temperature volatility and output in England over the 1800-2015 period. Kotz et al. (2021) examine data on over one thousand subnational regions over 40 years and show that day-to-day temperature variability reduces annual growth rates. Both analyses rely on *realized* volatility measures that correlate positively with extreme events, defined either as anomalies or large fluctuations in temperatures and precipitations. This correlation is inevitable because realized volatility is by construction dominated by large changes in temperatures or precipitations. It is also problematic from an identification perspective: if the relation between climate and growth is nonlinear, realized volatility may capture the abnormal impact of extreme events rather than a new, independent transmission mechanism. [3] We focus instead on an ex-ante volatility measure that is conceptually different from, and empirically unrelated to, the occurrence of extreme events. Furthermore, our econometric methodology allows us to describe jointly the evolution of first and second moments in the data, capturing various interactions between levels and volatilities. This reveals that temperature volatility affects macroeconomic volatility, uncovering a new channel through which climate change can affect welfare. [4]

The existence of a time-varying 'climate risk' factor is consistent with recent evidence obtained from firms and financial markets. Asset pricing models point to climate as an important risk factor in the long run (Bansal et al., 2016), and suggest that carbon risk and pollution are priced in the cross-section of stock returns (Bolton and Kacperczyk,

[2]See e.g. Bloom (2009), Jurado et al. (2015), Christiano et al. (2014), BANSAL et al. (2014). Extensive surveys are provided by Bloom (2014) and Fernández-Villaverde and Guerrón-Quintana (2020).

[3]Windstorms and hurricanes have significant economic consequences (Dell et al., 2014); recent research points to a generally nonlinear relation between temperatures and growth across countries and time periods (Burke et al., 2015).

[4]In treating climate as a stochastic rather than a deterministic system, our approach is also consistent with recent views in climatology, see e.g. Calel et al. (2020).

2020; see also Hong et al. (2020) and Giglio et al. (2020)). Furthermore, surveys and textual analyses of earning conference calls reveal that climate risk considerations feature prominently in the decisions of institutional investors (Krueger et al., 2020) and listed firms around the world (Sautner et al., 2020; Li et al., 2020). Our work complements these studies by constructing empirical measures of temperature volatility for a large panel of countries, documenting their historical patterns, and quantifying the macroeconomic implications of exogenous volatility shocks.

The paper is organized as follows. Section 2 describes the data and introduces our empirical model, a panel VAR with stochastic volatility. Section 3 illustrate the main empirical results. Section 4 explores various robustness checks and extensions of the baseline model. Section 5 concludes the paper.

2 Econometric methodology

We use the dataset assembled by Dell et al. (2012) (DJO). The data covers 133 countries and spans the years between 1961 and 2005. The weather data comes from the Terrestrial Air Temperature and Precipitation: 19002006 Gridded Monthly Time Series, which provides terrestrial monthly mean temperatures and precipitations at 0.5 0.5 degree resolution. DJO aggregate the series to the country-year level using population-weighted averages, with weights based on population in 1990. The GDP series are sourced from the World Development Indicators database maintained by the World Bank. We refer the reader to DJO for details on the definitions of the variables and descriptive statistics.

Our analysis has two related objectives. The first one is to estimate the conditional volatility of temperatures and precipitations for all countries in the sample. These estimates provide a clear and rigorous measure of climate predictability, and they allow us to assess whether predictability changed at all since the 1960s. The second one is to test whether climate volatility matters for economic growth, controlling of course for the influence of global warming documented elsewhere in the literature. To achieve these objectives in an internally-consistent fashion, we estimate a country-level panel VAR model where (i) temperatures and economic growth are linked by a two-way interaction; (ii) the residuals are heteroscedastic; and (iii) changes in volatility can have first-order effects on economic performance. The following subsections describe structure, identification and estimation of the model.

2.1 The panel VAR model

The panel VAR model with stochastic volatility has the following form:

$$Z_{it} = c_i + \tau_t + \sum_{j=1}^{P} \beta_j Z_{it-j} + \sum_{k=0}^{K} \gamma_k \tilde{h}_{it-k} + v_{it} \tag{1}$$

where Z_{it} is a vector of endogenous variables and countries and years are indexed by $i = 1, 2, ..., M$ and $t = 1, 2, ..., T$ respectively. The variance covariance matrix $cov\,(v_{it}) = \Omega_{it}$ is time-varying and heterogenous across countries. This matrix is factored as:

$$\Omega_{it} = A^{-1} H_{it} A^{-1'} \tag{2}$$

where A is a lower triangular matrix with ones on the main diagonal. H_{it} is a diagonal matrix $H_{it} = diag\left(\exp(\tilde{h}_{it})\right)$ where $\tilde{h}_{it} = [h_{1,it}, h_{2,it}, ..h_{N,it}]$ denotes the stochastic volatility of the orthogonalised shocks $\tilde{e}_{it} = Av_{it}$. The stochastic volatilities follow a panel VAR(1) process:

$$\tilde{h}_{it} = \alpha_i + \theta \tilde{h}_{it-1} + b_0 \tilde{\eta}_{it}, \tilde{\eta}_{it} \sim N(0, 1) \tag{3}$$

where α_i denotes country fixed effects, b_0 is a lower triangular matrix and $\tilde{\eta}_{it} = [\eta_{1,it}, \eta_{2,it}, ..., \eta_{N,it}]$ denotes a vector of volatility shocks.

The distinguishing feature of the model is that volatilities appear as regressors on the right-hand side of equation 1. Hence, if $\gamma_k \neq 0$, an exogenous increase in the volatility of any of the variables included in the model can affect the dynamics of the entire system. In our baseline specification we define $Z_{it} = [T_{it} \quad \Delta GDP_{it}]'$, where T_{it} denotes average annual temperature in degrees Celsius and ΔGDP_{it} is the annual growth rate of real GDP. To make the notation more intuitive, in the remainder of the paper we label the two level shocks (e^T_{it}, e^{GDP}_{it}), the volatility processes (h^T_{it}, h^{GDP}_{it}) and the associated volatility shocks (η^T_{it}, η^{GDP}_{it}). In this setup $e^T_{i,t}$ represents a shock to the temperature *level* T_{it}; h^T_{it} represents the log-volatility of e^T_{it}, i.e. of the (residual) portion of T_{it} that is unforecastable given past data; and η^T_{it} represents a temperature *volatility* shock, i.e. an exogenous shift in volatility occurring in country i at time t. We interpret h^T_{it} as an empirical measure of uncertainty about future temperatures and η^T_{it} as an unexpected temperature uncertainty shock. [5]

Intuitively, the model allows us to capture two mechanisms through which temper-

[5]In the robustness analysis we consider a range of alternative specifications that include e.g. annual precipitations or squared GDP and temperature changes, and distinguish *inter alia* between rich and poor countries.

ature volatility could affect growth. The first one is the direct impact of temperature volatility (h_{it}^T) on the growth rate of the economy ($\Delta GDP_{i,t}$). Higher volatility may reduce foreign investments, discourage risk-averse firms from undertaking new investment plans, or force them to engage in costly adaptation and insurance programs. The second one is a spillover from temperature volatility (h_{it}^T) to output volatility (h_{it}^{GDP}). To the extent that changes in temperatures matter for GDP growth rates, an increase in the frequency and/or magnitude of those changes could render growth more volatile. [6] The coexistence of these mechanisms implies that climate uncertainty could affect welfare in two ways, reducing an economy's average growth rate and rendering its behavior more erratic over time.

2.2 Identification and estimation

The growth regressions traditionally employed in the literature treat the climate system as an exogenous driving force. By contrast, our panel VAR model allows for a two-way interaction between climate and economic activity: in principle, T_{it} can affect growth and respond endogenously to changes in the level and volatility of ΔGDP_{it}. As in any VAR model, additional assumptions are thus needed in order to identify exogenous temperature shocks. Our key identification assumption is that macroeconomic developments have no contemporaneous (within-year) impact on climate variables. We apply this assumption to both level and volatility shocks by restricting the A^{-1} and b_0 matrices to be lower-triangular. In practice, this implies that e_{it}^{GDP} and η_{it}^{GDP} only affect T_{it} and h_{it}^T with a lag of at least one year. Recursive identification schemes are notoriously problematic when dealing with financial and macroeconomic data, but can be used safely in our application. Development and technological change may alter temperature and precipitation patterns over time, but this long-term phenomenon is very unlikely to have a material impact over a one-year horizon. Even if it did, our approach would approximate the data better than regression models that postulate strong(er) forms of exogeneity of the climate indicators.

The model is estimated using a Gibbs sampling algorithm that is described in detail in the technical appendix. The algorithm is an extension of methods used for Bayesian VARs with stochastic volatility (see e.g. Clark, 2011) to a panel setting. The parameters of the model can be collected into five blocks: $\left(\Gamma, \bar{A}, \bar{B}, Q, \tilde{h}_{it}\right)$. Here $B = vec\left([c_i, \tau_t, \beta_1, ..\beta_P, \gamma_1, ..\gamma_K]\right)$ denotes the coefficients of equation 1, $\bar{A}$ is a vector that collects the elements of A that are not equal to 0 or 1, $\bar{B} = vec\left([\alpha_i, \theta]\right)$ while $Q = b_0 b_0'$ is the variance of the residual of the transition equation (3). Each iteration of the algorithm samples from the conditional posterior distributions of these parameter blocks.

[6] Notice that this channel is entirely independent of the first one. If temperatures enter the production function, a volatility spillover could arise even in a linear, risk-neutral and frictionless economy where h^T has no direct influence on investment decisions.

Given $\tilde{h}_{it}$ and $\bar{A}$, the model is simply a panel VAR with a known form of heteroscedasticity. Therefore, given a normal prior, the conditional posterior of Γ is also normal after a GLS transformation. As described in Cogley and Sargent (2005), conditional on Γ and $\tilde{h}_{it}$, the elements of $\bar{A}$ are coefficients in linear regressions involving the residuals of the panel VAR. Therefore, their conditional posterior is standard. Given $\tilde{h}_{it}$, equation (3) is simply a panel VAR with fixed effects. As we employ conjugate priors for $\bar{B}$ and Q, their conditional posteriors are well known and easily sampled form. With a draw of $\Gamma, \bar{A}, \bar{B}, Q$ in hand, equations (1) to (3) constitute non-linear state space model for each country. To draw from the conditional posterior of $\tilde{h}_{it}$ we use the particle Gibbs sampler of Andrieu et al. (2010) and Lindsten et al. (2014). We use 55,000 iterations and retain every 10th draw after a burn-in period of 5000 draws. In the technical appendix we show that the estimated inefficiency factors are low, providing evidence in favor of convergence of the algorithm.

3 Empirical results

The model in Section 2 allows us to estimate the conditional volatility of annual temperatures at the country level between 1960 and 2005. These estimates offer a simple empirical characterization of short-term 'climate risk'. Conditional volatilities are intrinsically forward-looking: they capture the magnitude of the fluctuations in temperatures that are likely to materialize in each country at a given point in time. Unlike realized volatilities, they are not mechanically driven by the changes in temperatures observed in the recent past, including extreme events. And, as long as the data is persistent, they convey information on the likely evolution of the system: higher volatility signals the beginning of a (potentially long) phase of erratic weather conditions. Hence, our analysis captures a dimension of climate change and a transmission mechanism that could, at least in principle, operate alongside the traditional 'level' effect of rising temperatures documented elsewhere. In section 3.1 we discuss the evolution of temperature volatilities over time and its relation to the global warming phenomenon discussed in the literature. In section 3.2 we study the impact of exogenous changes in temperature volatility on economic growth.

3.1 Trends in temperature volatility

Figure 1 shows the behavior of temperatures and conditional temperature volatilities in six macro-regions between 1961 and 2005. Each region is summarized by a simple (unweighted) average of its member countries. The left panel replicates the findings of DJO: temperatures have increased across the board. The right panel shows that a similar trend occurred for volatilities too. Volatility has increased by 0.1 to 0.5°C depending on the region. Shifts of this magnitude could in principle have non-negligible economic implications. DJO estimate that a 1°C rise in annual temperatures reduce economic growth by over 1 percentage point on average in poor countries. Burke and Emerick (2016) find that temperature changes of -0.5 to +1.5 °C had a negative impact on agricultural output across US counties in the past, suggesting that even rich and technologically sophisticated economies may be vulnerable to climate fluctuations. The central question examined in this paper is whether an increase in the likelihood of facing larger temperature fluctuations in the future – i.e. an increase in the conditional volatility of annual temperatures – can have similar effects on growth (see Section 3.2).

The rise in volatility also occurred in Europe and Central Asia, which were instead only marginally affected by global warming. This divergence is interesting *per se* and it is useful for identification: if levels and volatilities were perfectly correlated it would be impossible to disentangle their effects. To investigate this point further, in Figure 2 we show the relation between temperature levels and volatilities at the country level. The scatter plot relates the cumulative change in temperatures recorded between 1961 and 2005 (horizontal axis) to the cumulative change in temperature volatility estimated by the model (vertical axis). There is no correlation between changes in levels and volatilities. In estimating the impact of volatility shocks in Section 3.2 we rely of course on a more stringent identification strategy; the model allows us to control country and/or time fixed effects as well as the lagged influence of GDP and temperatures. However, the lack of correlation in figure Figure 2 is reassuring because it indicates that the data contain sufficient information to separate level and volatility effects. The scatter plot also shows that the increase in volatility has been more widespread than the increase in average temperatures; in particular, many large economies experienced a rise in volatility combined with constant or decreasing average temperatures (see north-western quadrant).

Two further points are worth making. The changes in volatilities over time are highly significant from a statistical perspective. Figure A2 of the annex shows the average within-region volatility together with a 68% (one standard deviation) posterior coverage band. The null hypothesis that volatility did not change between the 1960s and the early 2000s can be safely rejected for all regions. In all but two cases, i.e. Sub-Saharan Africa and Eastern Europe, volatility follows a clear upward trend since the 1980s. At the same time, regional aggregations mask a significant degree of heterogeneity at the country level. In figure A3 we plot the confidence bands calculated at the regional level against the central estimates of the country-specific volatilities. It is immediately clear that level, variability and medium-run patterns in temperature volatility differ widely across countries, even within a given geographical region. The panel VAR employed below allows us to exploit cross-country heterogeneity in volatility patterns to identify the effects of exogenous changes in temperature volatility.

3.2 Temperature volatility and economic growth

The panel VAR introduced in Section 2 captures a number of interactions involving both the level and the volatility of annual temperatures and GDP growth at the country level. The posterior mean and standard deviation of the parameters for the baseline model is summarized in table A1 of the annex. The estimates highlight a potentially important influence of volatility on both GDP and temperatures: all else equal, a rise in temperature volatility h^T is associated with lower growth and higher temperatures. A rise in h^{GDP} is also associated with lower growth rates, which is consistent with the negative effects of economic volatility on investment highlighted in the literature. Finally, h^T and h^{GDP} are both highly persistent, suggesting that the sample is characterized by slow transitions between calm and volatile phases rather than sudden and short-lived outbursts of volatility.

In Figure 3 we report the impulse-response functions associated to the four shocks included in the model. For each variable the figure reports the estimated mean response with 90% and 68% posterior coverage bands. An exogenous increase in temperature volatility h^T causes an increase in temperatures, a decline in GDP growth and an increase in the volatility of GDP. The shock propagates through two distinct channels: h^T has (i) a direct negative impact on ΔGDP, and (ii) a positive impact on h^{GDP} , which is in itself detrimental for growth. This mechanism is quantitatively important. The one-standard-deviation shock examined in the figure raises the volatility of annual temperatures by about 1°C (row 1, column 3). This causes an increase in the volatility of GDP growth of about 1.3 percentage points (col. 4). This endogenous response indicates that countries that experience high temperature volatility in a given year are also likely to face significantly more pronounced fluctuations in GDP. In other words, higher temperature risks bring along higher risks for growth. The shock also generates a decline in GDP growth of about 0.8 percentage points on impact, with an economic slowdown that can last up to two years (col. 2). The GDP contraction results from the combined influence of higher climatic and economic volatility. It is worth emphasizing that the transmission mechanism hinges on risk rather than actual changes in temperature levels. First, $h^T_{i,t}$ measures by construction the conditional volatility of temperatures in country i, and it is unrelated to the weather patterns observed in i in the past. Second, the model controls for the influence of both contemporaneous and past temperature shocks on GDP growth. Third, the estimates remain unaltered if we include squared annual changes in temperatures or precipitations to control for the confounding influence of 'extreme' weather events (see Section 4). Hence, the results show that economic agents respond to the degree of expected variability of the environment, and that – as in the case of many other non-climatic factors – lower predictability is *per se* detrimental for growth.

The IRFs in Figure 3 also show that shocks to temperature levels and output raise respectively ΔT_t and ΔGDP_t (rows 2 and 4) in the year in which they occur, with negligible repercussions for the other variables in the system. The baseline specification suggests that a rise in temperatures has positive but non-significant effects on GDP, but the impact of the shock becomes negative if the model is estimated over poor countries only. Both results are consistent with those reported by DJO. Finally, an exogenous increase in h_t^{GDP} has a negative effect on ΔGDP_t. The GDP response is smaller and less persistent than that caused by a rise in h_t^T; this suggests that climate volatility is quantitatively more important than other sources of aggregate risk captured by the residuals of the growth equation.

The analysis carried out in this Section leads to two conclusions. The first one is that temperature volatility has risen steadily across countries since the 1960s, rendering climate conditions less predictable over time. The second one is that, controlling for temperature levels, a rise in temperature volatility is generally followed by a period of low and volatile GDP growth rates. Taken together, these findings point to two distinct mechanisms through which climate change could affect income and welfare over and above the well-known 'global warming' phenomenon. In the next section we examine the robustness of the baseline results along various dimensions, studying inter alia the role of precipitations, cross-country heterogeneity and the relation between volatility and extreme events.

4 Robustness and extensions

In this section we replicate the baseline analysis using alternative samples and model specifications. To save space we only discuss the impact of a temperature volatility shock on the level and conditional volatility of the GDP growth rate. The results of the tests are summarized in figures 4–5–6 and tables 1–2: the figures compare the responses to those obtained in the baseline model, while the tables report the estimated short-run and long-run impact of the shock along with %68 coverage intervals. [7]

There is an open debate on the cross-sectional and distributional implications of climate change: in particular, rising temperatures may affect only or mostly poor countries that rely heavily on agricolture and have limited adaptation capabilities (see Section 1).

[7] The annex to the paper provides a full set of impulse-responses for each of the specifications examined in this section.

The first issue we investigate is thus how the climate volatility channel identified by the baseline model varies across regions. We re-estimate the baseline panel VAR specifications using only data on "poor", "rich", "hot", or "non-agricultural" countries. All groups are defined using the dummy variables suggested by DJO. Poor is a dummy for countries that have below-median GDP per capita in their first year in the dataset, hot is dummy for countries with above-median average temperature in the 1950s, and agricultural is a dummy for countries with an above-median share of GDP in agriculture in 1995. The estimates are reported in Figure 4, which compares the impact on GDP of a one-standard deviation increase in temperature volatility obtained in the four alternative subsamples. The responses are qualitatively similar across samples: in all cases the shock causes growth to be lower and more volatile on impact and up to one year ahead. The differences across subsamples largely mirror those documented for temperature levels: in particular, rich and non-agricultural countries are relatively less affected than poor or hot countries. However, heterogeneity is less pronounced. Table 1 shows that the contemporaneous response of h_t^{GDP} is significant in all groups of countries and the contemporaneous response of ΔGDP_t is significant in all groups except rich countries. In the long term, the estimated impact of the shock on GDP becomes smaller for hot countries but larger and more significant for rich and non-agricultural countries (table 2). This suggests that rich countries smooth the impact of the shock rather than averting it altogether. All in all, the evidence indicates that climate *volatility* might be relevant even for highly-developed economies that can adjust efficiently to gradual changes in temperature *levels*.

As a next step, we extend the baseline specification to account for other factors that might affect the climate-growth nexus; the results are displayed in figure 5. Precipitations are often employed as a proxy of climatic change along with temperatures. We thus replicate the baseline analysis adding precipitation (P_{it}, in units of 100 mm per year) to temperatures and GDP growth. The identification of the shocks is again based on a recursive ordering of the model: and precipitations are ordered before GDP so to maintain the assumption that the climate is exogenous to macroeconomic shocks in the short term (see table 1). Section 2). Adding precipitations reduces the peak impact of a temperature volatility shock on both GDP and GDP volatility by roughly one third. The responses remain statistically significant, particularly in the short term (see table 1). Exogenous shifts in h^P have indeed qualitatively similar effects to h^T shocks, suggesting that the volatility channel operates through both temperatures and precipitations.

Another relevant issue is the potentially nonlinear nature of the relationship between climate and the economy (see Section 1). Examining this possibility is particularly important for our hypothesis because extreme events and volatility are naturally linked. In previous studies of the relation between climate volatility and growth, Donadelli et al.

(2020) and Kotz et al. (2021) have employed realized *(ex-post)* volatility measures that are by construction affected by the occurrence of large fluctuations in weather conditions. Donadelli et al. (2020) report a correlation between temperature volatility and extreme events – defined as rainfalls, floods, frosts, hot temperature anomalies and droughts – of 0.59 in the post-war period (see figure 1 of the paper). Kotz et al. (2021) capture temperature volatility using a day-to-day temperature variability indicator; the indicator is constructed using the intra-monthly standard deviation of daily temperatures, and then averaged over months to obtain an annual figure for each region. We find that in the pooled dataset this indicator has a correlation of 0.30 with the squared changes in annual temperatures taking place in the same year. Hence, although exploiting higher-frequency observations may ameliorate the problem, spikes in realized volatility may still refleft at least in part large increases in temperatures that took place during the year. If the effect of a change in temperatures on income is nonlinear, and the economy responds mostly or only to large shocks, then realized volatilities may correlate with GDP growth even if there is no direct causal link between the two. Our *ex-ante* measures of volatility are not subject to this limitation because they have no direct relation with past temperatures or precipitations. However, the identification problem may in principle arise in our case as well.[8] As a first check we examine the correlation between changes in temperature volatilities $h^T_{i,t}$ and squared annual changes in $T_{i,t}$, $P_{i,t}$ and $GDP_{i,t}$. These provide rough estimates of the realized annual volatility of the three series, capturing a range of 'extreme events' – i.e. large year-to-year shifts in temperatures, precipitations or income – that may potentially bias the estimation of the mechanism of interest. The correlations are extremely low for all geographical regions, which means that fluctuations in $h^T_{i,t}$ do not systematically overlap with large year-on-year changes in output or weather conditions (see figure A4). We then re-estimate the panel VAR adding to the baseline variables (i) squared GDP growth rates, (ii) squared temperatures, or (iii) the squared volatility terms. As figure 5 shows, none of these changes has major implications for our results. The initial (within-year) impact of the shock on GDP growth and GDP volatility is virtually unaffected by the inclusion of the squared terms. This corroborates the conclusion that the model picks up the specific influence of temperature volatility rather than nonlinearities involving GDP and temperature levels. For rich countries, the impact of the shocks is fact larger and more significant in case (iii) than in the baseline model: in particular, GDP growth drops by twice as much, both on impact (-0.8% *versus* -0.3%, table 1) and in the long term (-1.0% *versus* -0.6%, table 2).

[8] One reason behind this risk is that large spikes in temperatures are more likely to occur when volatility is high. Hence, even if they do not correlate with extreme events that took place in the past (as traditional realized volatility measures do), conditional volatilities could correlate with extreme events that will take place in the near future. In both cases a regression model could wrongly interpret the impact of those events as the result of a volatility shock.

Finally, we re-estimate the baseline model using only observations on the pre- or post-1980 period, or including a set of year×region fixed effects. As figure 6 shows, the results are again in line with the baseline estimates. It is interesting to note that the fixed effects reduce the estimated impact of the shock on GDP growth (from -0.8 to about -0.4 per cent) but have no influence on the response of GDP volatility. We can thus rule out the possibility that the results are driven by specific years or distorted by regional trends in temperatures or growth.

5 Conclusions

Rising temperatures are known to have a negative impact on economic growth, particularly in poor countries. This paper shows that climate change also affects economic outcomes through a volatility channel. We use a panel VAR model with stochastic volatility to identify exogenous changes in temperature volatility and assess their implications for the macroeconomy. We exploit the model to estimate the conditional volatility of annual temperatures for 133 countries between 1961 and 2005. These estimates capture the variability of the residual component of annual temperatures that cannot be predicted using past data, quantifying the *ex-ante* 'temperature risk' faced by households and firms in a given country at a given point in time. The model captures the interaction between levels and variances of annual temperatures and GDP growth rates, allowing the identification of exogenous temperature volatility shocks. The analysis yields two main conclusions. First, temperature volatility increased steadily over time, even in regions that were only marginally affected by global warming. Second, temperature volatility matters for growth. Changes in volatility affect both the means and the variances of the GDP growth rates of the countries in our sample. Controlling for temperature levels, a +1°C increase in volatility causes on average a -0.9 per cent decline in GDP growth and a +1.3 per cent increase in the volatility of the GDP growth rate. These mechanisms operate in rich, non-agricultural countries too, and they are statistically and economically significant even controlling for the influence of large realized fluctuations in GDP, temperatures or precipitations. These findings demonstrate that economic agents respond to changes in the expected variability of the environment. They also suggests that climate risk may have important *ex-ante* implications for welfare, as uncertainty has economic costs that can materialize before, and independently of, any observed change in temperatures.

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	GDP level			GDP volatility		
	16%	50%	84%	16%	50%	84%
Baseline	-1.260	-0.902	-0.546	1.066	1.305	1.518
Poor countries	-1.736	-1.245	-0.750	0.881	1.185	1.448
Hot countries	-1.473	-1.039	-0.604	0.770	1.030	1.292
Rich countries	-0.686	-0.315	0.013	0.184	0.484	0.785
Non-agricultural countries	-0.949	-0.533	-0.123	0.508	0.891	1.199
Pre-1980 sample	-1.521	-0.868	-0.044	0.357	0.601	0.821
Post-1980 sample	-0.789	-0.435	-0.098	1.027	1.210	1.392
Region-Year FEs	-0.828	-0.551	-0.253	1.109	1.286	1.446
Inc. precipitations	-0.806	-0.558	-0.312	0.618	0.795	0.974
Inc. squared GDP	-1.320	-0.959	-0.604	1.148	1.396	1.613
Inc. squared Temperatures	-1.286	-0.944	-0.592	1.080	1.310	1.529
Inc. squared volatilities	-2.295	-1.718	-1.134	1.165	1.356	1.533
Inc. squared volatilities, rich countries	-1.289	-0.783	-0.331	0.539	0.733	0.948

Table 1: SHORT-TERM IMPACT OF AN INCREASE IN TEMPERATURE VOLATILITY. Contemporaneous responses of annual GDP growth rates and GDP volatilities to an exogenous one-standard-deviation increase in the conditional volatility of annual temperatures ($h_{i,t}^T$). The rows refer to alternative specifications of the panel VAR model. The responses are measured in the year when the shock takes place. The table reports the median estimated response along with with the 16th and 84th percentile of the posterior distribution. The estimation sample includes 133 countries over the 1961-2005 period.

	GDP level			GDP volatility		
	16%	50%	84%	16%	50%	84%
Baseline	-1.802	-1.455	-1.075	1.110	1.361	1.575
Poor countries	-1.762	-1.352	-0.942	0.927	1.242	1.510
Hot countries	-1.302	-0.915	-0.544	0.822	1.089	1.347
Rich countries	-1.189	-0.631	-0.065	-0.257	0.364	0.838
Non-agricultural countries	-1.806	-1.207	-0.675	0.237	0.865	1.256
Pre-1980 sample	-1.626	-1.048	-0.392	0.209	0.532	0.794
Post-1980 sample	-1.358	-0.770	-0.163	1.077	1.301	1.512
Region-Year FEs	-0.831	-0.518	-0.210	1.200	1.380	1.536
Inc. precipitations	-0.370	-0.021	0.334	1.111	1.162	1.224
Inc. squared GDP	-1.708	-1.331	-0.956	1.196	1.451	1.665
Inc. squared temperatures	-0.972	-0.526	-0.071	1.128	1.368	1.588
Inc. squared volatilities	-2.360	-1.668	-0.860	1.213	1.419	1.603
Inc. squared volatilities, rich countries	-1.808	-1.024	-0.189	0.362	0.691	0.983

Table 2: LONG-TERM IMPACT OF AN INCREASE IN TEMPERATURE VOLATILITY. Long-run response of GDP levels and GDP volatilities to an exogenous one-standard-deviation increase in the conditional volatility of annual temperatures ($h^T_{i,t}$). The rows refer to alternative specification of the panel VAR model. All responses are measured 5 years after the materialization of the shock; the level response for GDP is calculated cumulating changes in growth rates over the horizon. The table reports the median estimated response along with with the 16th and 84th percentile of the posterior distribution. The estimation sample includes 133 countries over the 1961-2005 period.

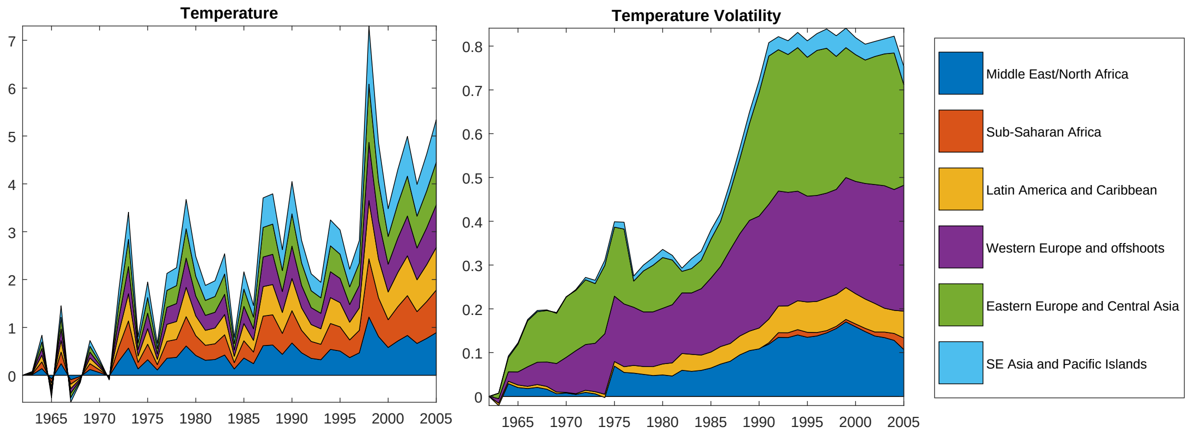

Figure 1: Trends in temperatures and volatility

The left panel shows the cumulative change in annual temperatures recorded between 1961 and 2005 in each of the six geographical regions listed in the legend. The right panel shows the cumulative change in each region's temperature volatility estimated by the panel VAR model. All figures are in degrees Celsius. The sample includes 133 countries and the regions are summarized by simple (unweighted) averages of country-level estimates.

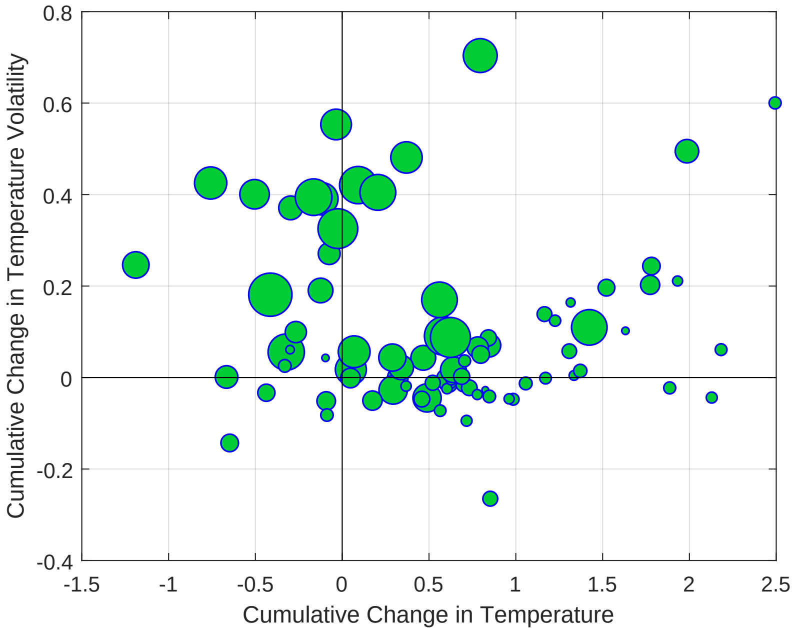

Figure 2: Correlation between temperature level and volatility

Total change in annual temperatures (horizontal axis) versus total change in estimated temperature volatilities (vertical axis). Temperatures and volatilities are in degrees Celsius. The sample includes 133 countries between 1961 and 2005. The size of the bubbles represents the countries' average GDP levels in the 1950-1959 period.

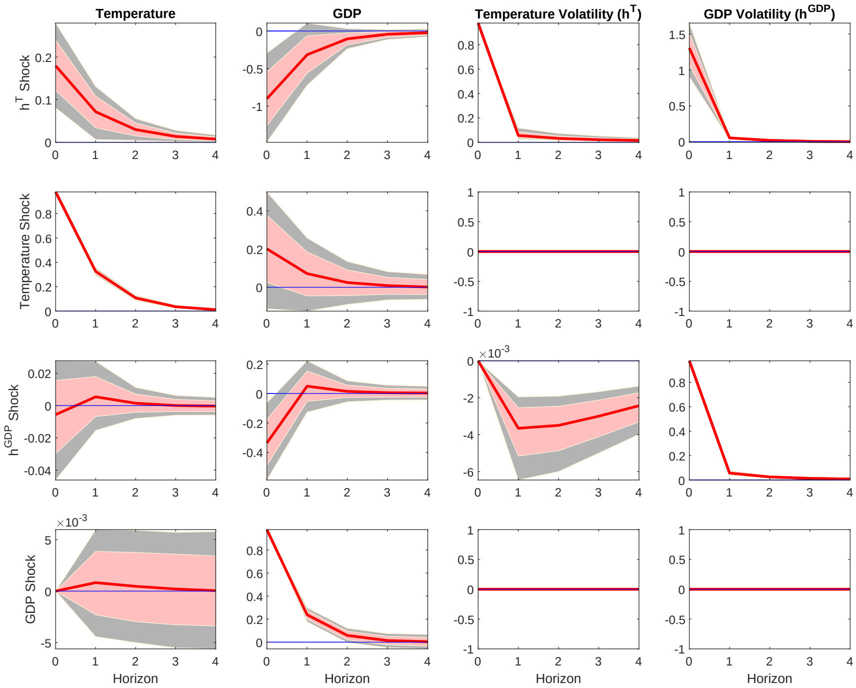

Figure 3: The propagation of level and volatility shocks

Impact of exogenous increases in either the level or the conditional volatility of annual temperatures and GDP in the baseline model. The estimates are obtained from a country-level panel VAR model where volatility is stochastic and changes in volatility influence the dynamics of all endogenous variables. *Temperature* and *GDP* are average annual temperature and annual GDP growth rate; $h^{T(GDP)}$ denotes the estimated conditional volatility of the temperature (GDP) series. The rows show the mean responses to a one-standard-deviation increase in, respectively, h^T, $Temperature$, h^{GDP} and GDP, with 68% and 90% posterior coverage bands. The estimation sample includes 133 countries between 1961 and 2005.

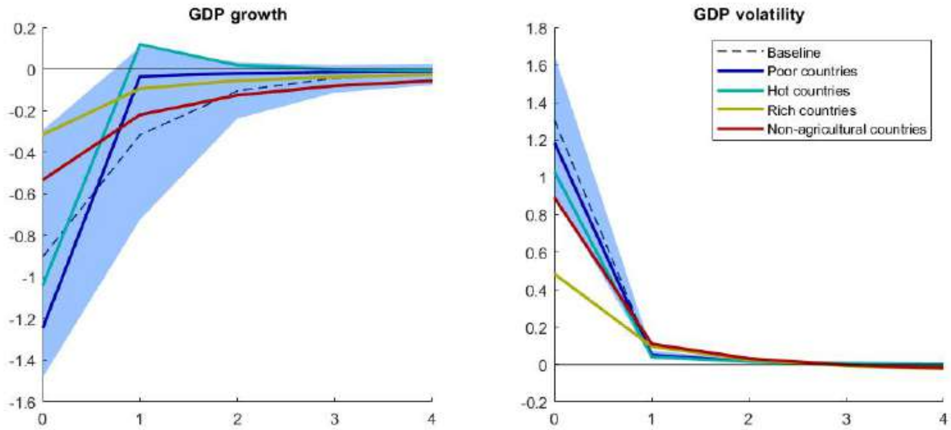

Figure 4: Heterogeneity across countries

Impact of a one-standard-deviation increase in temperature volatility on the annual growth rate of GDP (left panel) and its conditional volatility (right panel). The shaded area is the 90% confidence band obtained from the baseline panel VAR model. The additional lines represent the central estimates obtained restricting the estimation to a subsample of poor, hot, rich or non-agricoltural countries.

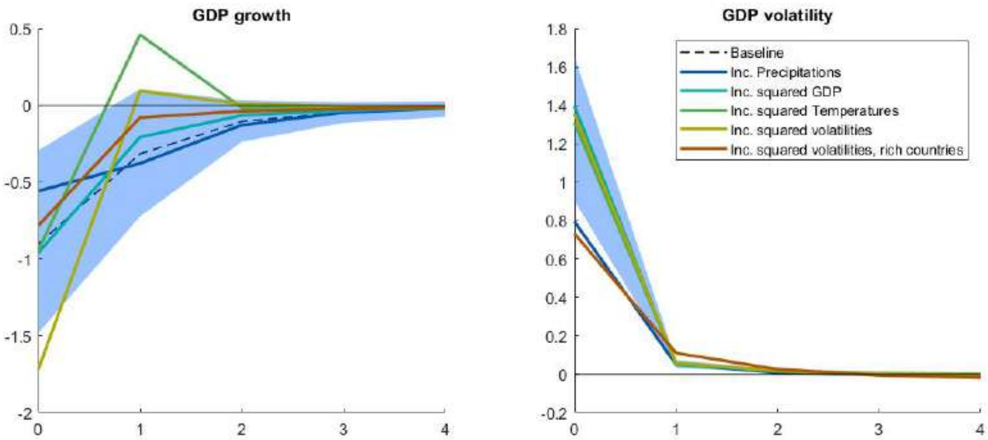

Figure 5: Additional controls

Impact of a one-standard deviation increase in temperature volatility on the annual growth rate of GDP (left panel) and its conditional volatility (right panel). The shaded area is the 90% confidence band obtained from the baseline panel VAR model. The additional lines represent the central estimates obtained by alternatively including in the model average yearly precipitations, squared GDP growth rates, squared temperatures or the squared volatilities of both GDP growth and temperatures. The latter specification is also estimated separately for rich countries.

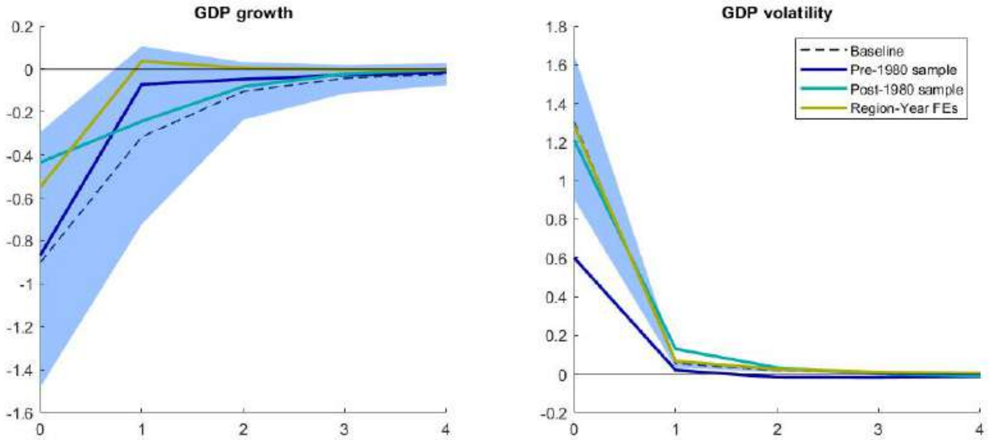

Figure 6: Subsamples and fixed effects

Impact of a one-standard deviation increase in temperature volatility on the annual growth rate of GDP (left panel) and its volatility (right panel). The shaded area is the 90% confidence band obtained from the baseline panel VAR model. The additional lines represent the central estimates obtained respectively estimating the model on the pre- and post-1980 period, or including a set of region-by-year fixed effects.

The Macroeconomic Cost of Climate Volatility

Online Appendix

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A.I Model Estimation

The panel VAR with stochastic volatility in mean is defined as:

$$\begin{aligned}
Z_{it} &= c_i + \tau_t + \sum_{j=1}^{P} \beta_j Z_{it-j} + \sum_{k=0}^{K} b_k \tilde{h}_{it-k} + \Omega_{it}^{1/2} e_{it}, e_t \sim N(0,1) && \text{(A.I.1)}\\
\Omega_{it} &= A^{-1} H_{it} A^{-1'} && \text{(A.I.2)}\\
\tilde{h}_{it} &= \alpha_i + \theta \tilde{h}_{it-1} + \eta_{it}, \eta_{it} \sim N(0,Q), && \text{(A.I.3)}\\
E\left(e_{it}, \eta_{it}\right) &= 0, E\left(e_{it}, e_{jt}\right) = 0, E\left(\eta_{it}, \eta_{jt}\right) = 0 \text{ for } j \neq i && \text{(A.I.4)}
\end{aligned}$$

Cross sections are indexed by $i = 1, 2, ..M$ while the time dimension is indexed by $t = 1, 2, .., T$. Equation A.I.1 describes the observation equation of the system, where Z_{it} is a $N \times 1$ matrix of endogenous variables, c_i and τ_t denote cross-section and time fixed effects. The vector $\underbrace{\tilde{h}_{it}}_{N\times 1}$ collects the stochastic volatilities of the orthogonalised shocks that are included as additional regressors: $\tilde{h}_{it} = [h_{1,it}, h_{2,it}, ..h_{N,it}]$. In equation A.I.2 $\underbrace{A}_{N\times N}$ denotes a lower triangular matrix with ones on the main diagonal and $H_{it} = diag\left(\exp\left(\tilde{h}_{it}\right)\right)$. The transition equation of the model is given by equation A.I.3 where α_i are cross-section fixed effects. The stochastic volatilties are assumed to follow a VAR(1) process and both θ and Q are full $N \times N$ matrices.

A.I.1 Prior distributions and starting values

VAR coefficients

Let $\Gamma = vec\left([\beta_j; b_k; c_i, \tau_t]\right)$ and denote the number of regressors excluding the lags of Z_{it} as EX. Note that the country and time-fixed effects are introduced into the model using dummy variables. Following Banbura et al. (2007), we employ a Normal prior implemented via dummy observations. The priors are implemented by the dummy observations y_D and x_D that are defined as:

$$y_D = \begin{bmatrix} \frac{diag(\gamma_1 s_1 \ldots \gamma_n s_n)}{\kappa} \\ 0_{N\times(P-1)\times N} \\ \ldots\ldots\ldots\ldots \\ 0_{EX\times N} \end{bmatrix}, \qquad x_D = \begin{bmatrix} \frac{J_P \otimes diag(s_1 \ldots s_n)}{\kappa} & 0_{NP\times EX} \\ \multicolumn{2}{c}{0_{N\times(NP)+EX}} \\ \multicolumn{2}{c}{\ldots\ldots\ldots\ldots} \\ 0_{EX\times NP} & I_{EX}\times 1/c \end{bmatrix} \qquad \text{(A.I.5)}$$

where γ_1 to γ_n denote the prior mean for the parameters on the first lag obtained by estimating individual AR(1) regressions, κ measures the tightness of the prior on the autoregressive VAR coefficients, and c is the tightness of the prior on the remaining regressors. We set $\tau = 1$ and $c = 1000$.

The priors for the coefficients are thus: $N\left(\Gamma_0, P_0\right)$ where $\Gamma_0 = \left(x_D' x_D\right)^{-1}\left(x_D' y_D\right)$ and $P_0 = S \otimes \left(x_D' x_D\right)^{-1}$ where S is a diagonal matrix with an estimate of the variance of Z_t.

Elements of H_{it}

To set initial values for the elements of H_{it}, we estimate a VAR with stochastic volatility for each country and obtain an intial estimate of the stochastic volatilities, denoted by μ_{it}. The prior for $\tilde{h}_{it}$ at $t = 0$ is defined as $\ln h_{i0} \sim N(\ln \mu_{i1}, I)$.

Elements of A

Using the lags of μ_{it} in equation A.I.1, we estimate the equation $Z_{it} = c_i + \tau_t + \sum_{j=1}^{P} \beta_j Z_{it-j} + \sum_{k=0}^{K} b_k u_{it-k} + v_{it}$ by OLS and obtain an intial estimate of the residuals of the VAR v_{it}. The prior for the off-diagonal elements A is $A_0 \sim N\left(\hat{a}^{ols}, V\left(\hat{a}^{ols}\right)\right)$ where $\hat{a}^{ols}$ are the off-diagonal elements of the inverse of the Cholesky decomposition of $var\left(v_{it}\right)$ where each row of the decomposition is divided by the corresponding element on the diagonal. $V\left(\hat{a}^{ols}\right)$ is assumed to be diagonal with the elements set equal to 1.

Parameters of the transition equation

The prior on the coefficients of the transition equation A.I.3 is implemented via dummy variables (see Banbura et al. (2007)), shrinking each equation towards an AR process. The prior tightness parameter controlling the strength of the prior on the coefficients on the lagged volatilities is set equal to 0.2. The prior for Q is inverse Wishart: $IW\left(Q_0, T_0\right)$ where $Q_0 = diag\left(g_0 \times T_0\right)$ with g_0 representing a vector that contains the average variance of the shocks to the transition equation obtained by the initial estimation of the VAR with stochastic volatility for each country. The degrees of freedom T_0 are set equal to $N + 1$

A.I.2 Simulating the conditional posterior distributions

VAR coefficients

Conditional on all other parameters, the model in equation A.I.1 is a panel VAR with a known form of heteroscedasticity. The conditional posterior distribution is normal: $N\left(\Gamma_{T\backslash T}, P_{T\backslash T}\right)$.

The model in A.I.1 can be written at each time period t as:

$$y_{it} = x_{it}\Gamma_t + \bar{e}_{it}$$

where:

$$\begin{aligned} y_{it} &= vec(Z_{it}) \\ x_{it} &= \begin{pmatrix} X_{1,t} \\ . \\ . \\ X_{M,t} \end{pmatrix} \\ X_{it} &= I_N \otimes \bar{x} \end{aligned}$$

$\bar{x}$ denotes all the RHS variables in equation A.I.1 at time t for country i. The variance of the error term $\bar{e}_{it}$ is:

$$var\left(\bar{e}_{it}\right) = R_t = blkdiag\left(\left[A^{-1}H_{1t}^{1/2}A^{-1\prime}, .., A^{-1}H_{Mt}^{1/2}A^{-1\prime}\right)$$

Finally, we assume that the transition equation for Γ_t is $\Gamma_t = \Gamma_{t-1}$. These equations form a conditionally linear and Gaussian state-space system. Following Carter and Kohn (1994) we use the Kalman filter to calculate the mean and the variance of the conditional posterior distribution of Γ. The Kalman filter is initialised at Γ_0 and P_0 and the recursions are given by the following equations for $t = 1, 2..T$

$$\begin{aligned}
\Gamma_{t\backslash t-1} &= \Gamma_{t-1\backslash t-1} \\
P_{t\backslash t-1} &= P_{t-1\backslash t-1} \\
\eta_{t\backslash t-1} &= y_t - x_t\Gamma_{t\backslash t-1} \\
f_{t\backslash t-1} &= x_t P_{t\backslash t-1} x_t' + R_t \\
K_t &= P_{t\backslash t-1} x_t' f_{t\backslash t-1}^{-1} \\
\Gamma_{t\backslash t} &= \Gamma_{t\backslash t-1} + K_t \eta_{t\backslash t-1} \\
P_{t\backslash t} &= P_{t\backslash t-1} - K_t x_t P_{t\backslash t-1}
\end{aligned}$$

The final iteration of the Kalman filter at time T delivers $\Gamma_{T\backslash T}$ and $P_{T\backslash T}$, the mean and the variance of the conditional posterior. This application of the Carter and Kohn (1994) algorithm to this heteroscedastic VAR model is equivalent to a GLS transformation of the model.

Element of A

Given a draw for Γ and $\tilde{h}_{it}$ the VAR model can be written as $A'\left(v_{it}\right) = \tilde{e}_{it}$ where $v_{it} = Z_t - (c_i + \tau_t + \sum_{j=1}^{P}\beta_j Z_{it-j} + \sum_{k=0}^{K} b_k \tilde{h}_{it-k})$ and $VAR\left(\tilde{e}_{it}\right) = H_{it}$. This is a system of linear equations with a known form of heteroscedasticity. The conditional distributions for a linear regression apply to each equation of this system after a simple GLS transformation to make the errors homoscedastic. The kth equation of this system is given as $v_{it}^k = -\alpha v_{it}^{-k} + \tilde{e}_{it}$ where the superscript k denotes the kth column of the residual matrix while $-k$ denotes columns 1 to $k-1$. Note that the variance of $\tilde{e}_{it}$ is time-varying and given by $\exp\left(\tilde{h}_{it}\right)$. A GLS transformation involves dividing both sides of the equation by $\sqrt{\exp\left(\tilde{h}_{it}\right)}$ to produce $v_{it}^{k*} = -\alpha v_{it}^{-k*} + e_{it}^*$ where $*$ denotes the transformed variables and $var\left(e_{it}^*\right) = 1$. The conditional posterior for α is normal with mean and variance given by M^* and V^* :

$$\begin{aligned}
M^* &= \left(V\left(\hat{a}^{ols}\right)^{-1} + v_{it}^{-k*\prime} v_{it}^{-k*}\right)^{-1}\left(V\left(\hat{a}^{ols}\right)^{-1}\hat{a}^{ols} + v_{it}^{-k*\prime} v_{it}^{k*}\right) \\
V^* &= \left(V\left(\hat{a}^{ols}\right)^{-1} + v_{it}^{-k*\prime} v_{it}^{-k*}\right)^{-1}
\end{aligned}$$

Elements of H_t

Conditional on the VAR coefficients and the parameters of the transition equation, the model has a multivariate non-linear state-space representation for each cross-section given by equations A.I.1 and A.I.2. Following recent developments in the seminal paper by Andrieu et al. (2010), we employ a particle Gibbs step to sample from the conditional posterior of $\tilde{h}_{it}$. Andrieu et al. (2010) show how a version of the particle filter, conditioned on a fixed trajectory for one of the particles can be used to produce draws that result in a Markov Kernel with a target distribution that is invariant. However, the usual problem of path degeneracy in the particle filter can result in poor mixing in the original version of particle Gibbs. Recent development, however, suggest that small modifications of this algorithm can largely alleviate this problem. In particular, Lindsten et al. (2014) propose the addition of a step that involves sampling the 'ancestors' or indices associated with the particle that is being conditioned on. They show that this results in a substantial improvement in the mixing of the algorithm even with a few particles.[1]As explained in Lindsten et al. (2014), ancestor sampling breaks the reference path into pieces and this causes the particle system to collapse towards something different than the reference path. In the absence of this step, the particle system tends to collapse to the conditioning path. We employ particle Gibbs with ancestor sampling in this step to draw $\tilde{h}_{it}$ for $i = 1.2, .., M$.

Let $\tilde{h}_{it}^{(d-1)}$ denote the fixed the fixed trajectory, for $t = 1, 2, ..T$ obtained in the previous draw of the Gibbs algorithm for country i. We denote the parameters of the model by Ξ, and $j = 1, 2, ..S$ represents the particles. The conditional particle filter with ancestor sampling proceeds in the steps described below. We suppress the cross-section index i in $\tilde{h}_{it}$ to keep the notation simple. In other words $\tilde{h}_t$ refers to the stochastic volatility for the ith cross-section. The steps described below are repeated for $i = 1, 2, .., M$.

1. For $t = 1$

   (a) Draw $\tilde{h}_1^{(j)} \backslash \tilde{h}_0^{(j)}, \Xi$ for $j = 1, 2, ..S-1$. Fix $\tilde{h}_1^{(S)} = \tilde{h}_1^{(d-1)}$

   (b) Compute the normalised weights $p_1^{(j)} = \frac{w_1^{(j)}}{\sum_{j=1}^{M} w_1^{(j)}}$ where $w_1^{(j)}$ denotes the conditional likelihood: $\left|\Omega_{i1}^{(j)}\right|^{-0.5} - 0.5 \exp\left(\tilde{e}_{i1}\left(\Omega_{i1}^{(j)}\right)^{-1}\tilde{e}_{i1}'\right)$ where $\tilde{e}_{i1} = Z_{it} - \left(c_i + \tau_t + \sum_{j=1}^{P}\beta_j Z_{it-j} + \sum_{k=1}^{K} b_k \tilde{h}_{1,[-k]}^{(j)}\right)$ and $\Omega_{i1}^{(j)} = A^{-1}H_{i1}^{(j)}A^{-1\prime}$ with $H_{i1}^{(j)} = diag\left(\exp\left(\tilde{h}_{1,[0]}^{(j)}\right)\right)$. The subscript [0] denotes the contemporaneous value in the state vector while $[-k]$ denote the k lagged states.

2. For $t = 2$ to T

   (a) Resample $\tilde{h}_{t-1}^{(j)}$ for $j = 1, 2, ..S-1$ using indices $a_t^{(j)}$ with $\Pr\left(a_t^{(j)} = j\right) \propto p_{t-1}^{(j)}$

   (b) Draw $\tilde{h}_t^{(j)} \backslash \tilde{h}_{t-1}^{(a_t^{(j)})}, \Xi$ for $j = 1, 2, ..S-1$ using the transition equation of the model (equation A.I.3). Note that $\tilde{h}_{t-1}^{(a_t^{(j)})}$ denotes the resampled particles in step (a) above.

   (c) Fix $\tilde{h}_t^{(S)} = \tilde{h}_t^{(d-1)}$

[1]See Nonejad (2016) for a recent application of this algorithm.

(d) Sample $a_t^{(M)}$ with $\Pr\left(a_t^{(M)}=j\right) \propto p_{t-1}^{(j)} \Pr\left(\tilde{h}_t^{(d-1)} \backslash \tilde{h}_{t-1}^{(j)}, \alpha, \theta, Q\right)$ where the density $\Pr\left(\tilde{h}_t^{(d-1)} \backslash \tilde{h}_{t-1}^{(j)}, \alpha, \theta, Q\right)$ is computed as $|Q|^{-0.5} - 0.5\exp\left(\tilde{\eta}_{it}^{(j)} (Q)^{-1} \tilde{\eta}_{it}^{(j)}\right)$, where $\tilde{\eta}_{it} = \tilde{h}_t^{(d-1)} - \left(\alpha + \theta \tilde{h}_{t-1}^{(j)}\right)$. This constitutes the ancestor sampling step. If $a_t^{(M)} = M$ then the algorithm collapses to the simple particle Gibbs.

(e) Update the weights $p_t^{(j)} = \frac{w_t^{(j)}}{\sum_{j=1}^{M} w_t^{(j)}}$ where $w_1^{(j)}$ denotes the conditional likelihood: $\left|\Omega_{it}^{(j)}\right|^{-0.5} - 0.5\exp\left(\tilde{e}_{it}\left(\Omega_{it}^{(j)}\right)^{-1}\tilde{e}_{it}'\right)$, where
$\tilde{e}_{it} = Z_{it} - \left(c_i + \tau_t + \sum_{j=1}^{P} \beta_j Z_{it-j} + \sum_{k=1}^{K} b_k \tilde{h}_{t,[-k]}^{(j)}\right)$,
$\Omega_{it}^{(j)} = A^{-1} H_{it}^{(j)} A^{-1\prime}$, with $H_{it}^{(j)} = diag\left(\exp\left(\tilde{h}_{t,[0]}^{(j)}\right)\right)$.

3. End

4. Sample $\tilde{h}_t^{(i)}$ with $\Pr\left(\tilde{h}_t^{(i)} = \tilde{h}_t^{(j)}\right) \propto p_T^{(j)}$ to obtain a draw from the conditional posterior distribution

We use $M = 20$ particles in our application. The initial values μ_0 defined above are used to initialise step 1 of the filter.

Parameters of the transition equation

Conditional on the draw for the volatilities, the conditional posterior for $\bar{B} = vec([\alpha_i, \theta])$ the parameters of the VAR in equation A.I.2 is Normal. Letting y and x denote the left and the right hand side of the VAR in A.I.2, the conditional posterior of the coefficients is defined as

$$G\left(\bar{B}\backslash\Xi\right) \tilde{} N(B^*, Q \otimes (x^{*\prime}x^*)^{-1})$$

where $B^* = (x^{*\prime}x^*)^{-1}(x^{*\prime}y^*)$ and x^* and y^* denote x and y appended with dummy observations.

The conditional posterior for Q is inverse Wishart and is given by

$$G(Q\backslash\Xi) \tilde{} IW(S^*, T^*)$$

where $T^* = MT + T_0$ with MT the total number of observations in the stacked data y and the scale matrix is $S^* = \left(y - x\bar{b}\right)'\left(y - x\bar{b}\right) + Q_0$.

The MCMC algorithm is applied using 55,000 iterations discarding the first 5,000 as burn-in and retaining every 10th draw. Figure A1 shows that the estimated inefficiency factors are fairly low. This provides evidence in favour of convergence.

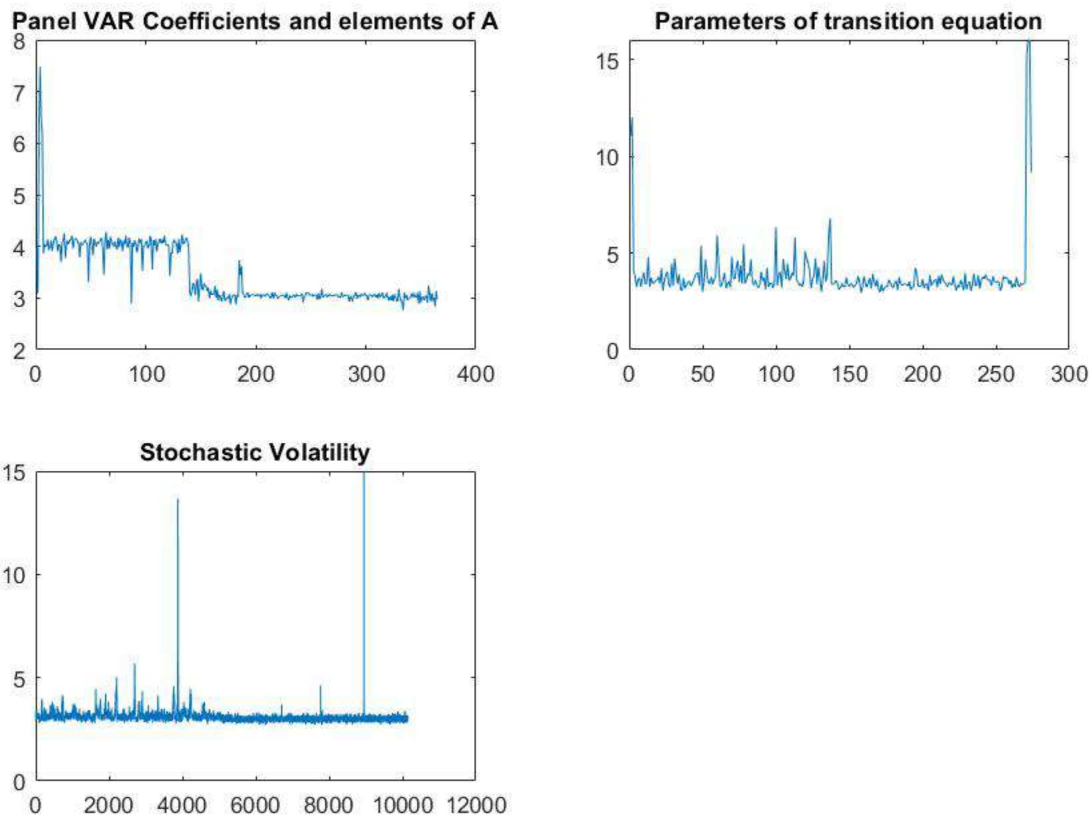

Figure A1: **Inefficiency factors**

A.II Supplementary Results

	$T_{i,t}$	$\Delta GDP_{i,t}$
$T_{i,t-1}$	0.333	0.023
	0.019	0.122
$\Delta GDP_{i,t-1}$	0.001	0.243
	0.001	0.019
$h_{i,t}^{T}$	0.196	-0.480
	0.076	0.439
$h_{i,t}^{GDP}$	-0.007	-0.338
	0.024	0.221
$h_{i,t-1}^{T}$	-0.014	-0.252
	0.073	0.447
$h_{i,t-1}^{GDP}$	0.010	0.152
	0.021	0.187
$e_{i,t}^{T}$	1	0.200
	-	0.471
$e_{i,t}^{GDP}$	0	1
	-	-
	h_t^T	h_t^{GDP}
h_{t-1}^{T}	0.899	-0.201
	0.014	0.033
h_{t-1}^{GDP}	-0.050	0.747
	0.011	0.026
η_t^T	1	1.323
	-	0.239
η_t^{GDP}	0	1
	-	-

Table A1: Baseline panel VAR estimates. The top panel reports posterior means and standard deviations of the parameters of the panel VAR model described in equation (A.I.1). T and GDP are average annual temperature in degrees Celsius and real GDP in country i and year t; $h^{T(GDP)}$ is the estimated conditional volatility of each country's temperature (GDP growth) in a given year; $e^{T(GDP)}$ represents a shock to temperatures (GDP growth) in the same year. The bottom panel reports the estimates for the autoregressive volatility process described in equation (A.I.2). $\eta^{T(GDP)}$ is a temperature (GDP growth) volatility shock. The estimation sample includes 133 countries over the 1961-2005 period.

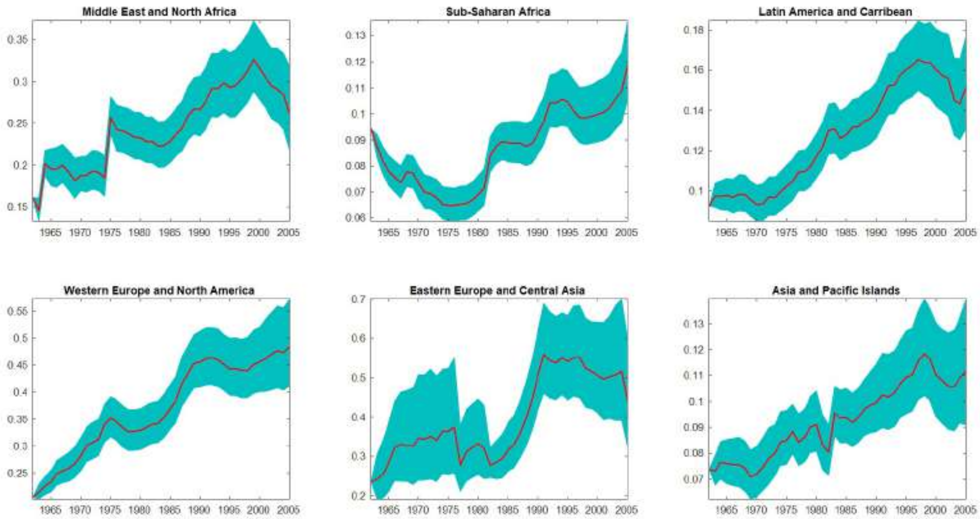

Figure A2: **Volatility trends across geographical regions**

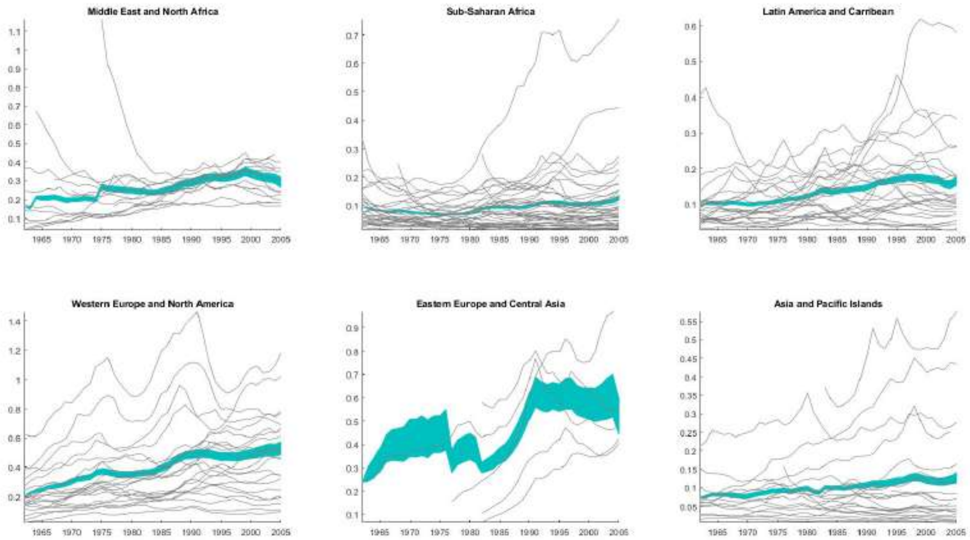

Figure A3: **Country-level volatility estimates**

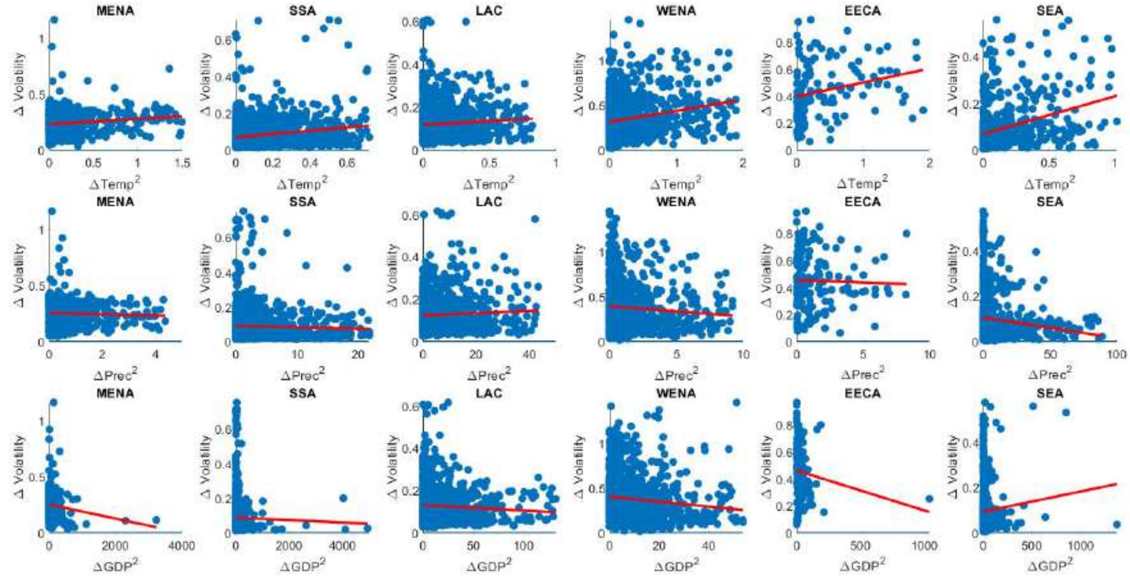

Figure A4: **Volatility versus extreme events.** The figure shows the country-level correlations between annual changes in temperature volatility (vertical axis) and 'extreme events' (horizontal axis), proxied alternatively by squared annual changes in temperatures (row 1), precipitations (row 2) or GDP (row 3). The countries are grouped into six regions: Middle East and North Africa, South-Saharan Africa, Latin America and Caribbean, Western Europe and North America, Eastern Europe and Central Asia, South-East Asia. The sample includes 133 countries between 1961 and 2005.

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