Disagreements in Society over Linear vs. over Nonlinear Labour-Income Tax Schedules

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Disagreements in Society over Linear vs. over Nonlinear Labour-Income Tax Schedules*

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Abstract

I investigate numerically the following question: Given that individuals with different wages disagree over the optimal labour-income tax schedule, how would such disagreements be affected if society restricted itself to using linear tax schedules? I find that there would be (i) a decrease in disagreements within a large segment of the population at the top of the wage distribution, (ii) a sharp decrease in how much a very-high-wage individual disagrees with individuals whose wage is weakly above the median but sufficiently far below her wage, (iii) a sharp decrease in how much a zero-wage individual disagrees with a median-wage individual, and (iv) a decrease in how much a zero-wage individual disagrees with high-wage individuals.

Keywords: labour-income tax; linear tax; redistribution; conflict

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1 Introduction

Consider the following two observations. First, issues related to redistribution are highly controversial and generate much conflict in society–for example, in the form of expensive lobbying and election campaigns, time spent on political discourse, and even violent revolutions. Second, laws and social norms regarding the permissible kinds of redistribution are likely to affect the level of conflict.

These observations motivate the following question: How would conflict in a society be affected if the society restricted itself, via social norms or (perhaps, constitutional) laws, to perform labour-income-based redistribution using solely linear tax schedules?[1] Such tax schedules hold considerable appeal for some economists and policy-makers, mainly on grounds of simplicity and the resulting benefits in terms of compliance costs, tax avoidance, and tax evasion.[2] However, the implications for conflict are, to the best of my knowledge, unexplored.[3] Although it falls well short of a satisfactory analysis of these implications, the current paper offers some (hopefully) relevant insights.

I consider two societies, Society L and Society N, each of which has to choose a tax schedule over labour income. The two societies are identical, except for the following difference. In Society L, the tax schedule is restricted to be linear. In Society N, the tax schedule can be piecewise linear with two pieces, subject to the constraint that it is weakly progressive in the sense of a weakly increasing average tax rate.

[1]Following terminology commonly used in public economics, I allow "linear" tax schedules to have a nonzero intercept although it would be more precise to call such tax schedules "affine".

[2]For a forceful case in favour of a "flat tax" as well as numerous examples of countries that have adopted a flat tax, see Hall and Rabushka (1995). (Hall and Rabushka's flat tax is characterised by a zero tax liability up to some exemption level and a tax liability that increases linearly for incomes above the exemption level. Thus, although quite similar to a linear tax, it isn't quite linear. Still, many of their arguments carry over to linear tax schedules.)

[3]Hall and Rabushka (1995) do claim that their flat tax would eliminate lobbying for loopholes. But that has to do with the fact that their proposal rules out loopholes rather than with the shape of their proposed tax schedule.

My analysis of the two societies proceeds as follows. Given two wages w and $\hat{w}$, I define a measure of how much type w (i.e., an individual with wage w) disagrees with type $\hat{w}$. This measure equals type w's gain (in dollars) if she, rather than type $\hat{w}$, gets to be the "dictator" choosing the tax schedule. Then, I numerically compute this measure for various $(w, \hat{w})$ pairs in Societies L and N. The computations (i) use a distribution of wages calibrated to the empirical distribution of wages in the United States and (ii) are performed separately for several empirically plausible values of the Hicksian wage elasticity of labour supply.[4]

The main findings are as follows. First, within a large segment of the population at the top of the wage distribution, there is complete agreement in Society L that the tax schedule should have the lowest possible marginal rate and a zero demogrant. As a result, disagreement within this segment of the population is larger in Society N.

Second, a very high type, w, disagrees much more in Society N than in Society L with types that are weakly above the median type but sufficiently far below w. The reason is that, in the role of a dictator in Society N (but not in Society L), any one of these types can extract revenue from type w without raising her own tax liability. As a result, type w fares much worse under such a dictator in Society N than in Society L.

Third, type 0 disagrees with the median type much more in Society N than in Society L. The reason is that a median-type dictator in Society N can extract revenue from types above her without having to share the proceeds with low types via a positive demogrant. As a result, type 0 fares much worse under a median-type dictator in Society N than in Society L.

Fourth, type 0 disagrees with high types more in Society N than in Society L. The reason is that, in Society N, type 0 obtains zero consumption under a high-type dictator (because the latter sets a zero demogrant) while, as a dictator herself, type

[4] A general theoretical analysis proved too difficult.

0 has more flexibility than in Society L to maximise the demogrant.

The paper proceeds as follows. Section 2 introduces the set-up. Section 3 introduces the measure of disagreement. Section 4 presents the calibration of the model. Section 5 contains the results. Section 6 discusses the limitations of the analysis.

2 Set-Up

2.1 Preferences

There is a unit mass of individuals who work and consume during a single period. I shall think of this period as being one year. All individuals have an identical preference over consumption, $c \geq 0$, (measured in dollars) and labour, $l \geq 0$, (measured in hours) that is represented by the utility function $u(c, l) = ce^{-\beta\epsilon l^{1/\epsilon}}$, where $\beta > 0$ and $\epsilon > 0$ is the Hicksian wage elasticity of labour supply. For this preference, the Marshallian wage elasticity of labour supply is zero.[5]

Note that $u(c, l)$ has the property $u(u(c, l), 0) = u(c, l)$, i.e., $u(c, l)$ gives the level of consumption such that the individual is indifferent between that level of consumption combined with zero labour and the bundle (c, l). Thus, u measures utility in dollar units. This will play a role in the measure of disagreement in section 3.

2.2 Types

Each individual has a wage (or type) $w \geq 0$ that is private information. Types are distributed according to a cumulative density function (CDF), F. Type w supplying labour l (measured in hours) earns pretax labour income wl. (Unless otherwise specified, "income" will refer to "pretax labour income" from now on.)

[5] It is more common to represent this same preference via the utility function $\hat{u}(c, l) = \ln(u(c, l)) = \ln(c) - \beta\epsilon l^{1/\epsilon}$. I use u because of the property in the next paragraph as well as because it is defined for $c = 0$.

2.3 Tax Schedules

Society has to choose a tax schedule over income, τ, $\tau(y)$ being the tax owed by an individual earning income y. The tax schedule has to generate enough revenue to finance an exogenously given level of government expenditures $G \geq 0$.

I consider two societies, Society L and Society N, that are identical except for the set of permissible tax schedules. In Society L, the tax schedule has to be of the form

$$\tau(y) = -t_0 + t_1 y, \tag{1}$$

where $t_0 \geq 0$ is the demogrant.

In Society N, the tax schedule has to be of the following piecewise linear form with (at most) two pieces:

$$\tau(y) = \begin{cases} -t_0 + t_1 y & \text{if } y \leq \hat{y} \\ -t_0 + t_1 \hat{y} + t_2(y - \hat{y}) & \text{if } y > \hat{y} \end{cases}, \tag{2}$$

where $t_0 \geq 0$ is the demogrant, $t_1 < 1$ and $t_2 < 1$ are the marginal rates on the first and second piece, respectively, $\hat{y}$ is the breakpoint, and $t_0 + (t_1 - t_2)\hat{y} \leq 0$. The latter constraint is equivalent to the requirement that the average tax rate be weakly increasing. This requirement reflects the idea that, in most modern societies, average-rate regressive tax schedules are plausibly ruled out from the outset by prevailing norms of fairness.

3 A Measure of Disagreement

In this section, I define a measure of how much an arbitrary type disagrees with another arbitrary type regarding which tax schedule should be chosen. This measure will be based on types' optimal tax schedules so that I start by defining these.

Let $l_w(\tau)$ denote type w's optimal labour supply given tax schedule τ.[6] In Society L (Society N, respectively), type w's optimal tax schedule solves

$$\max_{\tau} u(wl_w(\tau) - \tau(wl_w(\tau)), l_w(\tau)) \tag{3}$$
$$\int \tau(\tilde{w} l_{\tilde{w}}(\tau)) dF(\tilde{w}) \geq G$$
$$\tau \text{ is of the form in (1) (in (2), respectively).}$$

In words, type w's optimal tax schedule maximises type w's utility subject to the government's budget constraint, the requirement that the tax schedule is of the relevant form, and the requirement that each type is choosing her labour supply optimally.[7]

Let τ_w^X denote type w's optimal tax schedule in Society X $\in$ {N,L}.[8] Define $U^X(w, \hat{w}) = u\left(wl_w(\tau_{\hat{w}}^X) - \tau_{\hat{w}}^X(wl_w(\tau_{\hat{w}}^X)), l_w(\tau_{\hat{w}}^X)\right)$, i.e., $U^X(w, \hat{w})$ is type w's indirect utility (in dollars) in Society X if type $\hat{w}$ is a dictator setting the tax schedule. I define the measure of how much type w disagrees with type $\hat{w}$ in Society X as

$$\delta^X(w, \hat{w}) = U^X(w, w) - U^X(w, \hat{w}).$$

[6]In Society N, it is possible that a type w has two optimal levels of labour supply, which occurs if this type has an indifference curve that is tangent to both pieces of the tax schedule. In that case, I assume $l_w(\tau)$ equals type w's higher optimal labour supply. This assumption is most likely inconsequential given that said indifference can occur for at most a single type. Also, in both societies, $l_0(\tau)$ is indeterminate when $t_0 = 0$ (because type 0's utility is identically equal to zero, regardless of her labour supply). However, in that case, $l_0(\tau)$ can be set arbitrarily because both type 0's utility and tax revenue from type 0 are independent of $l_0(\tau)$.

[7]Given that the analysis will be numerical, why not allow general nonlinear taxation in Society N? The reason is that types' optimal tax schedules are likely to involve bunching (see Brett and Weymark (2017)), in which case the standard optimisation techniques cannot be applied. This led me to use piecewise linear tax schedules. The restriction to two pieces is dictated by computational constraints.

[8]For any w for which I compute τ_w^L and τ_w^N in the numerical analysis, I solve problem (3) by initiating the numerical maximisation algorithm from ten different seeds. This allows me to verify that τ_w^L and τ_w^N are essentially unique. I write "essentially" because, in Society N, there are sometimes multiple optimal tax schedules. However, these tax schedules only differ over a range of incomes that no type chooses so that the difference between them is immaterial.

That is, $\delta^X(w, \hat{w})$ equals type w's gain (in dollars) if she, rather than type $\hat{w}$, gets to be the dictator.

The following decomposition will be useful:

$$\delta^N(w, \hat{w}) - \delta^L(w, \hat{w}) = \left(U^N(w, w) - U^L(w, w)\right) + \left(U^L(w, \hat{w}) - U^N(w, \hat{w})\right). \quad (4)$$

Note that the term in the first parentheses is nonnegative (because type w has more flexibility in setting the tax schedule in Society N). Thus, if the term in the second parentheses is positive, then $\delta^N(w, \hat{w}) > \delta^L(w, \hat{w})$.

4 Calibration

For the numerical analysis, I need to choose the Hicksian elasticity (ϵ), the wage distribution (F), the preference parameter β, and government expenditures (G).[9]

4.1 Elasticity of Labour Supply

There is considerable controversy in the literature on the appropriate values for the Marshallian and Hicksian elasticities of labour supply with respect to the wage.[10] Table 6 in Keane (2011) reports estimates of these elasticities for males from a wide range of studies. I separately consider $\epsilon \in \{0.1, 0.5, 1\}$, which roughly covers the range of estimates of the Hicksian elasticity in that table.[11]

[9] In what follows, all dollar amounts are in 2012 dollars.

[10] Keane (2011) and Saez et al. (2012) provide surveys of this literature.

[11] In Keane's Table 6, the reported estimates of the Hicksian elasticities range between 0.02 and 1.32. Only two of the around two dozen Hicksian-elasticity estimates are above one. The reported estimates of the Marshallian elasticities range between -0.47 and 0.7, with an average value of 0.06. I decided against using more general preferences that allow for a nonzero Marshallian elasticity for two reasons. First, the average Marshallian elasticity of 0.06 is close to zero. Second, after plotting in a scatter plot all pairs of estimates (Hicksian elasticity, Marshallian elasticity) based on the studies in Keane's Table 6, I could not discern a natural and parsimonious way to cover all these pairs.

4.2 Wage Distribution

First, I obtain from Heathcote et. al (2023) an empirical distribution of hourly wages for individuals between the ages of 25 and 60 in the United States in 2021.[12]

Next, I approximate this empirical distribution via a CDF, $\hat{F}$, with 35 types in its support as follows. Let

$$\begin{aligned}(q_1, \ldots, q_{36}) = &(0, 7.5, 12.5, 17.5, 22.5, 27.5, 32.5, 37.5, 42.5, 47.5, 52.5, 57.5, 62.5, 67.5,\\ &72.5, 77.5, 82.5, 87.5, 88.5, 89.5, 90.5, 91.5, 92.5, 93.5, 94.5, 95.5, 96.5, 97.5,\\ &98.5, 99.5, 99.85, 99.95, 99.97, 99.99, 99.995, 100).\end{aligned}$$

I assume that type $i \in \{1, \ldots, 35\}$ equals the $\frac{q_i+q_{i+1}}{2}$-th percentile of the empirical distribution of hourly wages and has probability weight equal to $\frac{q_{i+1}-q_i}{100}$. For example, type 1 equals the 3.75-th percentile of the empirical distribution and has probability weight 0.075, type 2 equals the 10-th percentile of the empirical distribution and has probability weight 0.05, etc.

Finally, I obtain the CDF of types used in the analysis, F, by assuming that ten percent of the population have zero wage and rescaling the probabilities according to $\hat{F}$ of types $1, \ldots, 35$ (all of which have positive wages) by 0.9.[13] I add a zero-wage type because the empirical distribution of hourly wages excludes individuals who work zero hours, many of whom presumably would not work whatever the tax schedule.[14]

[12] Wages are computed as $\frac{\text{annual labour income + annual self-employment income}}{\text{annual hours worked}}$, where the denominator is based on the "new hours" measure, which the authors claim is superior. The computations exclude individuals who worked fewer than 260 hours in the year (the vast majority of these having worked zero hours so that wages cannot be computed). For details, see the definitions of labour income and self-employment income as well as the explanations for Figures 6 and 7 in Appendix A of their paper. Their paper itself reports insufficient data for my purposes, so I obtained the distribution of wages from the variable "new_wage" in the "cps_sampleC.dta" file provided in their replication materials.

[13] Some summary statistics of the wage distribution according to F are as follows: mean = 31; standard deviation = 39; (lowest wage, 50-th percentile, 90-th percentile, 99-th percentile, 99.9th percentile, highest wage) = (0, 24, 58, 173, 529, 1375).

[14] Nineteen percent of individuals in the data for 2021 in Heathcote et. al (2023) work zero hours.

4.3 The Parameter β

I set the parameter β as follows. I assume that the tax schedule individuals face in reality is of the form $\tau(y) = y - \lambda y^{0.818}$.[15] It is straightforward to show that, given this tax schedule, the optimal labour supply of any type $w > 0$ equals $\left(\frac{0.818}{\beta}\right)^{\epsilon}$. For each $\epsilon \in \{0.1, 0.5, 1\}$, I set β as the solution to $\left(\frac{0.818}{\beta}\right)^{\epsilon} = 2034$, where the number on the right-hand side is the average hours worked in 2021 by the individuals who were used to obtain the empirical distribution of hourly wages.[16]

4.4 Government Consumption Per Capita

According to the World Inequality Database, US national income per individual over age 20 in 2021 was $70,067.[17] According to Piketty, Saez, and Zucman (2018), total (i.e., federal, state, and local) government consumption in the US has been around 18 percent of national income since the end of World War II. I assume that (i) the labour share in national income is sixty percent[18] and (ii) the share of government expenditures financed from taxes on labour income equals the labour share in national income. Thus, I set $G = 70,067 \times 0.18 \times 0.6 \approx \$7,567$.

5 Results

In this section, I present the results of the numerical computations for $\epsilon = 0.5$. The results for $\epsilon = 0.1$ and $\epsilon = 1$ are very similar and are presented in the appendix.

These individuals do not work given the actual tax schedule they are facing. I am effectively assuming that around half of them would not work given any tax schedule, and I am ignoring the other half.

[15]This is the tax schedule over income (overall income, not just labour income) estimated by Heathcote et al. (2017) for the United States. λ is a constant that does not affect types' labour supply.

[16]The average hours worked is based on the "new hours" measure in Heathcote et. al (2023).

[17]The number is $80,860 in 2021 dollars, which I converted into 2012 dollars using the Bureau of Labor Statistics inflation calculator at https://data.bls.gov/cgi-bin/cpicalc.pl

[18]The Federal Reserve Bank of St. Louis reports on its website that the share of labour compensation in GDP was 0.597 in 2019. See https://fred.stlouisfed.org/series/LABSHPUSA156NRUG

	Society L		Society N			
	t_0	t_1	t_0	t_1	$\hat{y}$	t_2
0	24,244	0.73	25,638	0.87	23,690	0.64
w_{50}	17,420	0.43	0	-0.12	43,627	0.61
w_{90}	0	0.11	0	0.06	125,634	0.6
w_{99}	0	0.11	0	0.1	386,409	0.57
$w_{99.9}$	0	0.11	0	0.11	1,189,282	0.44

Table 1: Representative types' optimal tax schedules in Societies L and N.

Let w_p denote the p-th percentile of F. Much of the analysis below focuses on disagreements between the following types which represent different parts of the wage distribution: 0, w_{50}, w_{90}, w_{99}, and $w_{99.9}$.

Table 1 shows the representative types' optimal tax schedules in Societies L and N. Note that, in Society L, the high types prefer a zero demogrant and a marginal tax rate that is only just high enough to cover the government expenditures, G. In fact, there is complete agreement about this among a large fraction of the population at the top of the wage distribution.

Finding 1 $\delta^L(w, \hat{w}) = 0$ *for any two types in the top 38.25 percent of types.*

In Society N, type 0 (which seeks to maximise the demogrant) prefers a very high marginal rate up to \$23,690 and a lower (but still high) marginal rate above that income level. Each other representative type prefers a low (possibly, even negative) marginal rate up to a given level of income, which happens to be precisely her level of income, and a much higher marginal rate above that level of income.

Table 2 shows $U^N(w, w) - U^L(w, w)$ and $U^L(w, \hat{w}) - U^N(w, \hat{w})$ for various combinations of representative types. Given the decomposition in (4), $\delta^N(w, \hat{w}) - \delta^L(w, \hat{w})$ can readily be computed from Table 2. Nevertheless, I also explicitly present $\delta^N(w, \hat{w}) - \delta^L(w, \hat{w})$ for the various combinations of representative types in Table 3.

In light of Finding 1, it is unsurprising that $\delta^N(w, \hat{w}) > \delta^L(w, \hat{w})$ for any $w, \hat{w} \in$

	$U^N(w,w) - U^L(w,w)$	$U^L(w,\hat{w}) - U^N(w,\hat{w})$				
		$\hat{w}=0$	$\hat{w}=w_{50}$	$\hat{w}=w_{90}$	$\hat{w}=w_{99}$	$\hat{w}=w_{99.9}$
$w=0$	1,394	-	17,420	0	0	0
$w=w_{50}$	4,838	1,338	-	-1,434	-360	-21
$w=w_{90}$	3,393	-3,752	3,148	-	-871	-51
$w=w_{99}$	2,603	-19,145	33,031	70,793	-	-152
$w=w_{99.9}$	463	-64,896	121,494	310,637	210,961	-

Table 2: $U^N(w,w) - U^L(w,w)$ and $U^L(w,\hat{w}) - U^N(w,\hat{w})$ for various combinations of representative types. To avoid clutter, the diagonal entries in the right part of the table are omitted.

	$\hat{w}=0$	$\hat{w}=w_{50}$	$\hat{w}=w_{90}$	$\hat{w}=w_{99}$	$\hat{w}=w_{99.9}$
$w=0$	0	18,815	1,394	1,394	1,394
$w=w_{50}$	6,176	0	3,404	4,477	4,817
$w=w_{90}$	-359	6,541	0	2,522	3,343
$w=w_{99}$	-16,541	35,634	73,396	0	2,452
$w=w_{99.9}$	-64,433	121,957	311,100	211,425	0

Table 3: $\delta^N(w,\hat{w}) - \delta^L(w,\hat{w})$ for various combinations of representative types.

$\{w_{90}, w_{99}, w_{99.9}\}$ such that $w \neq \hat{w}$. The following findings highlight other interesting aspects of the results in Tables 2 and 3.

Finding 2 *For any* $w \in \{w_{99}, w_{99.9}\}$ *and* $\hat{w} \in \{w_{50}, w_{90}, w_{99}\}$ *such that* $w > \hat{w}$, $U^L(w,\hat{w}) \gg U^N(w,\hat{w})$ *and, hence,* $\delta^N(w,\hat{w}) \gg \delta^L(w,\hat{w})$.[19]

The reasons for the $U^L(w,\hat{w}) \gg U^N(w,\hat{w})$ part are the following: (i) unlike in Society N, in Society L a dictator of type $\hat{w} \in \{w_{50}, w_{90}, w_{99}\}$ has to pay the same marginal rate on her income as the one she imposes on incomes above hers, which dampens her optimal marginal rate,[20] (ii) as a result, under a type-$\hat{w}$ dictator, type $w \in \{w_{99}, w_{99.9}\}$ (where $w > \hat{w}$) has to pay a much higher marginal rate in Society

[19] "$\gg$" means "much larger than". Clearly, the "much" part involves some subjective judgement.

[20] This dampening effect in Society L is weaker for a below-median-wage dictator because her income is lower so that the tax rate she pays on it is less important to her. (She cares more about the demogrant.)

N than in Society L on the difference between her income and type $\hat{w}$'s income, and (iii) this difference is large given that types at the top of the income distribution are very spread out ($w_{99.9} \gg w_{99} \gg w_{90}$; see footnote 13).

Finding 3 *$U^L(0, w_{50}) \gg U^N(0, w_{50})$ and, hence, $\delta^N(0, w_{50}) \gg \delta^L(0, w_{50})$.*

The explanation for $U^L(0, w_{50}) \gg U^N(0, w_{50})$ goes as follows. In Society L, a type-w_{50} dictator chooses a substantial marginal rate because, although she is hurt by having to pay it herself, it also allows her to generate revenue from types above her, which benefits her through a large demogrant. This large demogrant also benefits type 0. In contrast, in Society N, a type-w_{50} dictator prefers not to use a positive demogrant, which would have to be shared with all other types. Instead she chooses a zero demogrant, combined with a low (in fact, negative) marginal rate at incomes up to her income level and a high marginal rate at incomes above that.

Remark 1: The logic in the previous paragraph continues to apply if type w_{50} is replaced by any positive, below-median type with the following caveat: in Society N, a dictator of a positive type sufficiently close to zero may opt for a large demogrant because it may not be worth it for her to use up one of the two pieces of the tax schedule by having a zero demogrant and a low marginal rate on the narrow range of incomes up to her level of income. However, this caveat is an artefact of the restriction to two-piece tax schedules. Thus, if more general nonlinear taxation were allowed in Society N, Finding 3 would probably continue to hold with any positive, below-median type replacing w_{50}.

Remark 2 (for future reference): In Society N, any above-median-type dictator is likely to set a zero demogrant for the same reasons for which a median-type dictator sets a zero demogrant.

Finding 4 *For $\hat{w} \in \{w_{90}, w_{99}, w_{99.9}\}$, $\delta^N(0, \hat{w}) > \delta^L(0, \hat{w})$.*

Given the decomposition in (4), the explanation for this finding is that (i) in both societies, type 0 gets zero consumption under a high-type dictator given that the latter prefers a zero demogrant (so that $U^L(0,\hat{w}) = U^N(0,\hat{w})$ for $\hat{w} \in \{w_{90}, w_{99}, w_{99.9}\}$) while (ii) type 0's optimal tax schedule involves a higher demogrant in Society N because the more flexible taxation makes it possible to more effectively extract revenue from higher types (so that $U^N(0,0) > U^L(0,0)$).

Remark 3: Let $\hat{w}$ be any type in the top 38.25 percent of types. Given that a type-$\hat{w}$ dictator sets a zero demogrant in Society L and given Remark 2, $U^L(0,\hat{w}) = U^N(0,\hat{w})$ probably holds. The latter equality and $U^N(0,0) > U^L(0,0)$ imply $\delta^N(0,\hat{w}) > \delta^L(0,\hat{w})$.

Remark 4: Let $\hat{w}$ be any type such that $w_{50} < \hat{w} \leq w_{0.6175}$. Given that a type-$\hat{w}$ dictator sets a positive demogrant in Society L and given Remark 2, $U^L(0,\hat{w}) > U^N(0,\hat{w})$ probably holds. The latter inequality and $U^N(0,0) > U^L(0,0)$ imply $\delta^N(0,\hat{w}) > \delta^L(0,\hat{w})$.

Putting together Remarks 3 and 4, Finding 4 probably continues to hold if $\hat{w}$ is any above-median type.

Finding 5 *For $w \in \{w_{99}, w_{99.9}\}$, $\delta^L(w,0) \gg \delta^N(w,0)$.*

The reason for this finding is as follows. Type 0's optimal tax schedule in Society N happens to involve a marginal rate at incomes above \$23,690 that is lower than her optimal marginal rate in Society L. It turns out that, for $w \in \{w_{99}, w_{99.9}\}$, the resulting negative value of $U^L(w,0) - U^N(w,0)$ is more than enough to offset the positive value of $U^N(w,w) - U^L(w,w)$.

6 Limitations

I conclude by discussing some important limitations of the analysis.

6.1 Assumptions behind the Findings

The findings are derived based on strong assumptions. First, the model of labour supply is a simple, static one in which all individuals have identical preferences over consumption and labour (with a constant Hicksian wage elasticity and a zero Marshallian wage elasticity).

Second, I rely on particular (albeit, hopefully, reasonably realistic) calibrations of the model. One aspect of these calibrations worth noting is that the empirical distribution on which F is based provides a snapshot of a single year. This is a limitation because (i) the chosen tax schedule is likely to be in place for multiple years (so that any disagreement over tax schedules would presumably be based on individuals' average wages over a multi-year period) and (ii) individuals' wages are imperfectly correlated across years (so that individuals' average wages over a multi-year period are likely to exhibit less dispersion than the empirical distribution of wages in a single year).

Third, there is an implicit assumption that Society N, with its two-piece tax schedules, can yield insights that extend to a society in which more general, average-rate progressive taxation is permissible.

Although these assumptions are strong, I suspect that the following continue to hold with greater generality:

(i) the finding that, within a large segment of the population at the top of the wage distribution, there is complete agreement in Society L (see Finding 1),

(ii) this finding's implication that disagreement within this segment of the population would be larger in a society that allows for nonlinear tax schedules (regardless of whether these are required to be piecewise linear with a given number of pieces, to be average-rate progressive, etc.),

(iii) Finding 2 given the large magnitudes of the relevant entries in Tables 2 and 3 and given that the key difference between Societies N and L behind it–namely, the ability of a type weakly above the median in Society N to extract revenue from types far above her without raising her own tax liability–applies more generally (i.e., regardless of the specific assumptions adopted here on preferences, distribution of types, and permissible tax schedules in Society N),

(iv) Finding 3 (as well as its extension in Remark 1) given the large magnitudes of the relevant entries in Tables 2 and 3 and given that the key difference between Societies N and L behind it–namely, the ability of a dictator of the median (or of a positive, below-median) type in Society N to extract revenue from types above her without having to share the proceeds with low types via a demogrant–applies more generally,[21] and

(v) Finding 4 (as well as its extensions in Remarks 3 and 4) given that the key factors behind it–namely, the incentive for an above-median-type dictator in Society N to set a zero demogrant and the ability of a type-0 dictator to more flexibly maximise the demogrant in Society N–probably apply more generally.

Other results, like Finding 5 and the entries in Tables 2 and 3 not addressed in Findings 1-5, are more likely to be an artefact of the particular assumptions made

[21]Finding 3 (in its current form but not in a form that replaces "$\gg$" with the more modest "$>$") also relies on the result that a median-type dictator sets a substantial demogrant in Society L. Although the latter result holds for the calibrations of the model considered here, it may not generalise. Still, it is plausible that, for a wide range of realistic calibrations of the model, even if a median-type dictator doesn't set a large demogrant in Society L, a w_p-type dictator would do so for some p less than 50 but not far below 50. Then, Finding 3 and Remark 1 would apply with w_p taking the role of w_{50}.

here.

6.2 Lack of Conclusions about Conflict

The findings about how much various types disagree with various other types do not allow us to draw conclusions about how much resources each type or society at large would spend on conflict (lobbying, rent-seeking, etc.) and, in particular, about how such expenditures would be affected if taxation were restricted to be linear.[22] This is a serious limitation of the analysis in the current paper. Nevertheless, perhaps the findings here provide a useful step.

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[22]In the appendix, I briefly discuss two approaches to getting at conflict–one based on measures of polarisation in a society and one based on explicitly modelling conflict through a game-theoretic approach. I also explain the difficulties of applying these approaches in the current context.

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	Society L		Society N			
	t_0	t_1	t_0	t_1	$\hat{y}$	t_2
0	40,656	0.92	41,814	0.98	28,292	0.87
w_{50}	36,148	0.76	0	-0.29	46,169	0.86
w_{90}	0	0.12	0	0.02	117,981	0.85
w_{99}	0	0.12	0	0.09	358,432	0.86
$w_{99.9}$	0	0.12	0	0.12	1,097,318	0.8

Table 4: Representative types' optimal tax schedules in Societies L and N given $\epsilon = 0.1$.

	Society L		Society N			
	t_0	t_1	t_0	t_1	$\hat{y}$	t_2
0	17,106	0.6	18,461	0.97	34,751	0.44
w_{50}	9,555	0.26	0	-0.07	43,297	0.46
w_{90}	0	0.1	0	0.07	137,253	0.43
w_{99}	0	0.1	0	0.09	426,340	0.4
$w_{99.9}$	0	0.1	0	0.1	1,315,003	0.29

Table 5: Representative types' optimal tax schedules in Societies L and N given $\epsilon = 1$.

7 Appendix A: Results for $\epsilon = 0.1$ and $\epsilon = 1$

Tables 4 and 5 are the counterparts of Table 1 but for $\epsilon = 0.1$ and $\epsilon = 1$, respectively. Finding 1 continues to hold for $\epsilon = 0.1$ and $\epsilon = 1$, except that for $\epsilon = 0.1$ the 38.25 number needs to be replaced with 33.75. It also continues to be the case that, apart from type 0, each representative type prefers a low (possibly, even negative) marginal rate up to a level of income, which happens to be precisely her level of income, and a much higher marginal rate above that level of income.

Tables 6 and 7 are the counterparts of Table 2 but for $\epsilon = 0.1$ and $\epsilon = 1$, respectively. Tables 8 and 9 are the counterparts of Table 3 but for $\epsilon = 0.1$ and $\epsilon = 1$, respectively. Findings 2-5 continue to hold, except that, in the case of $\epsilon = 0.1$, $\delta^L(w_{99}, 0) \gg \delta^N(w_{99}, 0)$ in Finding 5 may need to be tempered to $\delta^L(w_{99}, 0) > \delta^N(w_{99}, 0)$. Remarks 1-4 also continue to apply, except that, in the case of $\epsilon = 0.1$, the 38.25 number in Re-

	$U^N(w,w) - U^L(w,w)$	$U^L(w,\hat{w}) - U^N(w,\hat{w})$				
		$\hat{w}=0$	$\hat{w}=w_{50}$	$\hat{w}=w_{90}$	$\hat{w}=w_{99}$	$\hat{w}=w_{99.9}$
$w=0$	1,158	-	36,148	0	0	0
$w=w_{50}$	11,430	134	-	-4,246	-1,024	-80
$w=w_{90}$	10,164	-2,587	-6,033	-	-2,478	-194
$w=w_{99}$	7,428	-12,138	16,934	145,344	-	-583
$w=w_{99.9}$	1,782	-41,571	86,560	635,496	478,765	-

Table 6: $U^N(w,w) - U^L(w,w)$ and $U^L(w,\hat{w}) - U^N(w,\hat{w})$ for various combinations of representative types given $\epsilon = 0.1$. To avoid clutter, the diagonal entries in the right part of the table are omitted.

	$U^N(w,w) - U^L(w,w)$	$U^L(w,\hat{w}) - U^N(w,\hat{w})$				
		$\hat{w}=0$	$\hat{w}=w_{50}$	$\hat{w}=w_{90}$	$\hat{w}=w_{99}$	$\hat{w}=w_{99.9}$
$w=0$	1,355	-	9,555	0	0	0
$w=w_{50}$	2,258	-487	-	-571	-144	-7
$w=w_{90}$	1,335	-991	4,411	-	-349	-16
$w=w_{99}$	1,041	-18,334	26,426	32,397	-	-49
$w=w_{99.9}$	151	-69,837	91,941	141,276	93,659	-

Table 7: $U^N(w,w) - U^L(w,w)$ and $U^L(w,\hat{w}) - U^N(w,\hat{w})$ for various combinations of representative types given $\epsilon = 1$. To avoid clutter, the diagonal entries in the right part of the table are omitted.

	$\hat{w}=0$	$\hat{w}=w_{50}$	$\hat{w}=w_{90}$	$\hat{w}=w_{99}$	$\hat{w}=w_{99.9}$
$w=0$	0	37,306	1,158	1,158	1,158
$w=w_{50}$	11,564	0	7,184	10,406	11,350
$w=w_{90}$	7,577	4,131	0	7,686	9,970
$w=w_{99}$	-4,711	24,361	152,771	0	6,845
$w=w_{99.9}$	-39,789	88,342	637,278	480,547	0

Table 8: $\delta^N(w,\hat{w}) - \delta^L(w,\hat{w})$ for various combinations of representative types given $\epsilon = 0.1$.

	$\hat{w} = 0$	$\hat{w} = w_{50}$	$\hat{w} = w_{90}$	$\hat{w} = w_{99}$	$\hat{w} = w_{99.9}$
$w = 0$	0	10,910	1,355	1,355	1,355
$w = w_{50}$	1,771	0	1,687	2,114	2,251
$w = w_{90}$	344	5,746	0	986	1,318
$w = w_{99}$	-17,294	27,467	33,438	0	991
$w = w_{99.9}$	-69,686	92,092	141,428	93,810	0

Table 9: $\delta^N(w, \hat{w}) - \delta^L(w, \hat{w})$ for various combinations of representative types given $\epsilon = 1$.

mark 3 needs to be replaced with 33.75 and $w_{0.6175}$ in Remark 4 needs to be replaced with $w_{0.6625}$.

8 Appendix B: Measures of Polarisation and a Game-Theoretic Approach

8.1 Measures of Polarisation

Esteban and Ray (1994), Duclos et. al (2004), and Esteban and Ray (2011) introduce measures of polarisation in a society and provide axiomatic foundations for them. The axiomatisations are based on notions of "identification" and "alienation".

In the context of the current paper, the measures in Esteban and Ray (1994) and in the main body of Esteban and Ray (2011) can be applied to the 5×5 matrix with (i, j)-th entry $\delta^X(i\text{-th representative type}, j\text{-th representative type})$ (where X $\in$ {N,L}) or, perhaps, to a larger matrix that is based on a larger number of representative types. However, (i) the choice of representative types is arbitrary and (ii) the axiomatisations assume that alienation is symmetric, which is incompatible with the fact we can (and, typically, do) have $\delta^X(w, \hat{w}) \neq \delta^X(\hat{w}, w)$.[23]

[23] Section 5.2 in Esteban and Ray (1994) briefly discusses a special case of asymmetric alienation, but that special case does not apply here.

The polarisation measures in Duclos et. al (2004) and in the appendix to Esteban and Ray (2011) assume individuals differ along a continuous characteristic, which in the current context would be wages.[24] Thus, the need for arbitrary representative types can be avoided. However, the axiomatisations again assume that alienation is symmetric.[25] Even if we ignore this, (i) these measures need to be modified in order to apply them to the context of the current paper[26] and (ii) computing the modified measures would require computing the whole functions $\delta^L(\cdot,\cdot)$ and $\delta^N(\cdot,\cdot)$, which is computationally infeasible.

In addition, at a conceptual level, it is difficult to know what to make of axiomatisations which are based on "identification" and "alienation" given that these notions are not based on behaviour.[27]

8.2 A Game-Theoretic Approach

A large literature models conflict via a game-theoretic approach.[28] In this literature, individuals strategically expend resources on conflict, and these expenditures determine each individual's probability that she "wins". Typically, each individual cares about winning, but, if she doesn't win, she doesn't care about which other individual wins. Esteban and Ray (1999) and Esteban and Ray (2011) allow individuals who don't win to care about who does win. This extension is important from the perspective of the current paper because (i) here winning corresponds to becoming the dictator setting the tax schedule and (ii) types who aren't the dictator clearly care

[24] F would have to be replaced by a CDF that admits a probability density function.

[25] Section 2.4 in Duclos et. al (2004) briefly discusses a special case of asymmetric alienation, but that special case does not apply here.

[26] In particular, the absolute distances in these measures would need to be replaced by $\delta^L(\cdot,\cdot)$ and $\delta^N(\cdot,\cdot)$.

[27] Esteban and Ray (2011) link via numerical simulations a measure of polarisation to conflict based on the game-theoretic approach. However, their simulations assume a kind of symmetry which is incompatible with $\delta^X(w,\hat{w}) \neq \delta^X(\hat{w},w)$.

[28] For textbook treatments, see Konrad (2015) and Vojnović (2015).

about which type is the dictator.

Nevertheless, there are numerous difficulties of applying the approach of Esteban and Ray (1999) and Esteban and Ray (2011) to the current paper. First, these papers assume that individuals' preferences are quasilinear in the consumption good. In the current context, the lack of quasilinearity creates the following endogeneity which probably renders the analysis intractable. Expenditures on conflict would strategically depend on the types' labour supplies given each tax schedule as well as the types' optimal tax schedules. However, the types' labour supplies given each tax schedule and the types' optimal tax schedules would themselves depend on expenditures on conflict because of wealth effects.

Second, to perform numerical analysis, one needs to take a stand on the technology of conflict. However, there is little empirical basis for doing this, especially given that this technology is likely to be context-specific.

Third, one would need to take a stand on risk attitudes over consumption-labour bundles.

Fourth, Esteban and Ray (1999) and Esteban and Ray (2011) assume that individuals are grouped into a finite number of exogenously given groups and individuals in the same group have perfectly aligned preferences. In the current context, this creates three difficulties. First, given that different types can have different preferences over tax schedules, one would need to make some assumption about the preference over tax schedules of a group of types.[29] Second, one would need to make an assumption about the degree to which group members can free-ride on each other in the provision of resources for conflict. Third, one would need to make a largely arbitrary assumption about the exact exogenous partition of types into groups. A crucial complication here is that types would likely endogenously form different groups in Societies N and

[29] This is not insurmountable. For example, one could assume that the group's preference coincides with the preference of the median individual in it.

L. For example, it is plausible that a large fraction of the population at the top of the wage distribution would belong to the same group in Society L (given Finding 1) but not in Society N (given the disagreements at the top of the wage distribution in Tables 3, 8, and 9).

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